7. Explanation:

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ is cgt

D'Alembert Ratio test

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = l$

$l < 1$ cgt

$l > 1$ dgt

$l = 1$ test fail

Given series can be written as

$a_n = (-1)^{n+1}\frac{x^n}{n}$

$a_{n+1} = (-1)^{n+2}\frac{x^{n+1}}{n+1}$

Now $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$

$= \lim_{n\to\infty}\left|\frac{x_{n+1}}{n+1}\cdot\frac{n}{x^n}\right| < 1$

$= \lim_{n\to\infty}|x|\left|\frac{n}{n+1}\right| < 1$

$= |x| < 1$

This is cgt $\Rightarrow |x| < 1$

$\therefore$ option (3) is correct

8. Explanation:

(i) $u_n = \sqrt{n+1} - \sqrt{n}$

$\sum a_n = \sum\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} \times \sqrt{n+1} + \sqrt{n}$

$u_n = \sum\frac{(n+1)-n}{\sqrt{n+1}+\sqrt{n}}$

$u_n = \sum\frac{1}{\sqrt{n+1}+\sqrt{n}}$

$\sum \forall n = \sum\frac{1}{\sqrt{n}}\forall n = \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty}\frac{u_n}{vn} = \frac{1}{\sqrt{n+1}+\sqrt{n}} \times \frac{\sqrt{n}}{1}$

$= \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$

$= \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+1\right)}$

$= \frac{1}{2} \neq 0$

$\sum u_n$ & $\sum Vn$ has same behaviour

But $\sum Vn = \frac{1}{\sqrt{n}}$ is dgt

$\sum u_n$ is also dgt

(ii) $V_n = \sqrt{n^4+1} - n^2$

$\sum v_n = \sum\frac{\sqrt{n^4+1}-n^2}{\sqrt{n^4+1}+n^2} \times \sqrt{n^4+1} + n^2$

$v_n = \sum\frac{(n^4+1)-n^4}{\sqrt{n^4+1}+n^2} = \sum\frac{1}{\sqrt{n^4+1}+n^2}$

$\sum t_n = \sum\frac{1}{n^2}$ $\qquad t_n = 1/n^2$

$\lim_{n\to\infty}\frac{v_n}{t_n} = \lim_{n\to\infty}\frac{1}{\sqrt{n^4+1}+n^2} \times \frac{n^2}{1}$

$= \lim_{n\to\infty}\frac{n^2}{n^2\left(\sqrt{1+\frac{1}{n^4}}+1\right)}$

$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{2} \neq 0$

$\sum v_n$ & $\sum t_n$ have some behaviour

$\therefore \sum t_n = \sum\frac{1}{n^2}$ is cgt

$\therefore \sum v_n$ is cgt

$\Rightarrow$ option (2) is correct

9. Explanation:

The series $\sum_{n=1}^{\infty}\frac{n+1}{n^p}$ is convergent for p>2

10. Explanation:

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$

By Leibnitz test of alternating series $\{(-1)^{n+1}\}$ is bounded &

$\lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$

$\therefore \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$ is cgt

$\sum_{n=1}^{\infty}|a_n| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

We known that

$\sum\frac{1}{n^p}$ is $cgt\ P > 1$

$dgt\ p \leq 1$

11. Explanation:

$\sum a_n$ converges

Suppose $\sum\frac{1}{n^2}$ is converges

$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{1}{n^2}$

$\Rightarrow \lim_{n\to\infty} a_n = 0$

$\Rightarrow$ option (1) is correct

12. Explanation:

The radius of converges of the series

$1 + 2x + 3x^2 + 4x^3 + ...$

$1 + 2x + 3x^2 + 4x^3 + \cdots = \sum_{n=0}^{\infty}(n-1)x^n$

$= \frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

$= \lim_{n\to\infty}\left|\frac{n+1+1}{n+1}\right|$

$= \frac{1}{R} = \lim_{n\to\infty}\left|\frac{n+2}{n+1}\right|$

$= \frac{1}{R} = 1$

$R = 1$

$\Rightarrow$ option (2) is correct

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13. Explanation:

$\sum_{n=1}^{\infty} \frac{n^q+1}{n^p} = \sum_{n=1}^{\infty} \frac{n^q\left(1+\frac{1}{n^q}\right)}{n^p}$

$= \sum \frac{\left(1+\frac{1}{n^q}\right)}{n^{p-q}}$

Let

$\sum a_n = \sum \frac{1}{n^{p-q}}\left(1+\frac{1}{n^q}\right)$

And $\sum b_n = \sum \frac{1}{n^{p-q}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{n^{p-q}}\left(1+\frac{1}{n^q}\right).n^{p-q}$

$= \lim_{n\to\infty}\left(1+\frac{1}{n^q}\right) = 1 \neq 0$

But $\sum \frac{1}{n^{p-q}}$ is cgt

Iff $p-q>1$

⇒ option (D)

14. Explanation:

$a_n = \frac{(-1)^n n}{2^n}$

$\lim_{n\to\infty} a_n = 0$

As $2^n > n$

And $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n}{2^n} = 0$

$\therefore \sum_{n=1}^{\infty} a_n$ is absolutely convergent

$b_n = \frac{(-1)^n}{\log(n+1)}, |b_n| = \frac{1}{\log(n+1)}$

And we know that

$\log(n+1) < n+1$

$\frac{1}{\log(n+1)} > \frac{1}{n+1}$

And $\sum \frac{1}{(a_n+b)^p}$ is cg tiff $p>1$

$\therefore \sum \frac{1}{\log(n+1)}$ is dgt

But $\sum \frac{1}{\log(n+1)}$ is conditionally convergent by Leibnitz test

As $\sum(-1)^n a_n$

Where $a_n = \frac{1}{\log(n+1)}$

And $\lim_{n\to\infty} \frac{1}{\log(n+1)}$

15. Explanation:

(1) $\sum_{n=1}^{\infty} n^2$

Let $a_n = n^2$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2 = \infty$

⇒ $\sum_{n=1}^{\infty} a_n$ is divetgent

(2) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

$a_n = \frac{1}{n(n+1)} < \frac{1}{n^2}$

By direct compression test

$\sum \frac{1}{n^2}$ convergent.

⇒ $\sum \frac{1}{n(n+1)}$ is convergent.

(3) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

Here $a_n = \left(\frac{3}{2}\right)^n$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\left(\frac{3}{2}\right)^{n+1}}{\left(\frac{3}{2}\right)^n} = \frac{3}{2} > 1$

⇒ by ratio test

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$ *is divergent*

(4) $\sum_{n=1}^{\infty} n^3$

$\lim_{n\to\infty} n^3 = \infty$

⇒ $\sum_{n=1}^{\infty} n^3$ *is divergent*

16. Explanation:

Let $a_n = \frac{x^n}{n^n}$

$\frac{1}{R} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{x^{n+1}}{(n+1)^{n+1}} \times \frac{n^n}{x^n}$

$= \lim_{n\to\infty} \frac{x}{n+1} \times \left(\frac{n}{n+1}\right)^n$

$\frac{1}{R} = 0$

⇒ $R = \infty$

⇒ $\forall x \in \mathbb{R}$ the series converges

[PRACTICE QUESTIONS]

1. **Explanation:** The series $\sum \frac{1}{n(\log n)^p}$ is convergent if $p>1$ and divergent if $0 < p \leq 1$

⇒ option (1) is correct

2. **Explanation:** The set for convergence of an alternating series was given by Leibnitz test

⇒ Cauchy, D'Alembert, Raabe test for convergence of positive series

⇒ option (4) is correct

3. **Explanation:**

The infinite series

$1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ to ∞

We know

$\therefore \sum_{n=1}^{\infty} \frac{1}{n^p} = \begin{cases} p>1 & convergent \\ p \leq 1 & divergent \end{cases}$

⇒ series convergent if $P>1$

⇒ option (4) is correct.

4. **Explanation:**

The infinite series expansion of

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$\log(1+x) = x - \frac{x^2}{x} + \frac{x^3}{3} - \cdots$

$x = -1 \Rightarrow \log(1-1) = \log(0) = \infty$

$\therefore \log(1+x)$ is valid for $-1 < x \leq 1$

$\Rightarrow$ option (4) is correct

5. **Explanation:**

Given that, $a_n = b_n - b_{n+1} \forall\, n \in N$

Let $\langle S_n \rangle$ be the sequence of partial sans of $\sum a_n$, then

$S_n = a_1 + a_2 + .. + a_n$

$= (b_1 - b_2) + (b_2 - b_3) + \cdots + (b_n - b_{n+1})$

$= (b_1 - b_{n+1})$

Since, $\sum b_n$ is convergent, then $\lim_{n\to\infty} b_n = 0$

$\therefore \lim S_n = \lim(b_1 - b_{n+1}) = b_1 - 0 = b_1$

$\therefore \sum a_n$ is convergent and converges to b_1

Hence option (2) is correct.

6. **Explanation:**

Compare the infinite series, with

$\sum a_n y^n$, we have $a_n = \frac{1}{2^n}, y = x^3$

$\therefore \rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{2^{n+1}} 2^n = \frac{1}{2}$

$\therefore R = \frac{1}{\rho} = 2$

Hence series converges if $|y| < R$

i.e. $|x^3| < 2$ *i. e.* $|x| < 2^{1/3}$

Hence option (4) is correct

7. **Explanation:**

For option (C): Consider, $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n+1}$ both are divergent series,

But their sum $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty}\left(\frac{1}{n} + \frac{1}{n+1}\right)$ is also a divergent series, so option (C) is not true.

For option (B) Consider, $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(-1)^n$ and $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}(-1)^{n+1}$ both are divergent series (oscillatory)

But their sum $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty}((-1)^n + (-1)^{n+1}) = 0$ is convergent series, so option (B) is not true.

For option (D): Consider, $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent series and $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} n$ is divergent series.

But their $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty}\left(n + \frac{1}{n^2}\right)$ is divergent.

so, option (D) is not true.

For option (A): Given that infinite series, $\sum a_n$ diverges

We, consider the cases

Case-1: $a_n \geq 1$ for infinitely many $n \in \mathbb{N}$

$\therefore$ we have, for such $n \in \mathbb{N}$,

$\frac{a_n}{1+a_n} = 1 - \frac{1}{1+a_n} \geq 1 - \frac{1}{1+1} = \frac{1}{2}$

Since, $\sum_{n=1}^{\infty} \frac{1}{2}$ diverges, so by comparison test, $\sum_{n=1}^{\infty} \frac{a-n}{1+a_n}$ diverges

Case-2 $a_n < 1$ for infinitely many $n \in \mathbb{N}$

Then, for such we $n \in \mathbb{N}$, we have, $\frac{a_n}{1+a_n} \geq \frac{a_n}{1+1} = \frac{a_n}{2}$

Also, finitely many terms do not affect the convergence/divergence of a series,

Since, $\sum \frac{a_n}{2}$ diverges, so by comparison test, $\sum \frac{a_n}{1+a_n}$ diverges

Hence option (1) is correct

8. **Explanation:**

Note that, $x \leq |x| \forall\, x \in \mathbb{R}$

i.e. $-|x| \leq x \leq |\forall| x \in \mathbb{R}$

$\therefore$ for all $n \in \mathbb{N}$, we have, $-|a_n| \leq a_n \leq |a_n|$

$\Rightarrow -\sum_{n=1}^{\infty}|a - n| \leq \sum_{n=1}^{\infty} a_n \leq \sum_{n=1}^{\infty}|a_n|$

i.e. $-B \leq A \leq B$(as sum is finite)

i.e. $|A| \leq B$

Hence option (2) is correct.

9. **Explanation:**

For option (A):

Since, $\lim_{n\to\infty} \cos\left(\frac{1}{n^2}\right) = \cos 0 = 1 \neq 0$, Hence $\sum \cos\left(\frac{1}{n^2}\right)$ is divergent.

For option (B):

Let $v_n = \sin\left(\frac{1}{n^2}\right)$ and $u_n = \left(\frac{1}{n^2}\right)$. Then

$\lim_{n\to\infty} \frac{v_n}{u_n} = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)} = 1$

Which is non-zero and finite, hence $\sum u_n$, and $\sum v_n$ are identical in behaviour.

Since, $\sum u_n$ is convergent (by p-series test), So is $\sum v_n$.

For option (C):

We have $\sum v_n = \sum \frac{1}{n^{\left(1+\frac{1}{n}\right)}}$,

$v_n = \frac{1}{n^{\left(1+\frac{1}{n}\right)}} = \frac{1}{n.n^{\left(\frac{1}{n}\right)}}$

Let us assume that $u_n = \frac{1}{n}$,then $\frac{v_n}{v_n} = \lim_{n\to\infty} \frac{1}{n^{1/n}} = 1$ ()

$\left(\lim_{n\to\infty} n^{1/n} = 1\right)$ which is non-zero and finite.

Hence by limit comparison test, $\sum v_n$, and $\sum u_n$, are identical in behaviour.

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But $\sum v_n$, is divergent (by p-series test), so is $\sum u_n$

For option (D):

Comparing with $p-$series, we got $p = -cos3 < 1$

$(Also - \cos 3 > 0)$

Hence by p-series test, $\sum v_n$ is divergent

Hence option (2) is correct.

10. Explanation: (A) Note that fourier series expansion off $f(x) = x^2, -\pi \le x \le \pi$. Given as

$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$

Put $x = \pi$ we get,

$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n.(-1)^n}{n^2} \Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

Using this result, we proceed further as, $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

$\frac{\pi^2}{6} = \left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \cdots\right)$

$\frac{\pi^2}{6} = \sum_{k=1}^{\infty}\frac{1}{(2k-1)^2} + \frac{1}{4}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$

$\Rightarrow \sum_{k=1}^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$

Hence option (1) is correct.

11. Explanation:

$$x \in \left(0, \frac{1}{e}\right)$$

12. Explanation: The General term is given by

$u_k = \frac{k^2+3k+1}{(k+2)!} = \frac{(k+1)(k+2)-1}{(k+2)!}$

$= \frac{(k+1)(k+2)}{(k+2)!} - \frac{1}{(k+2)!}$

$= \frac{1}{k!} - \frac{1}{(k+2)!} \forall k$

Therefore $\sum_{k!}^{\infty} = \left(1 - \frac{1}{2!}\right) + \left(1 - \frac{1}{3!}\right) + \left(\frac{1}{2!} - \frac{1}{4!}\right) + \cdots$

$= 1 + 1 - 2$ (as rest term cancel out each other)

Hence option (3) is correct

13. Explanation: Given that,

$xe^x = \sum_{n=1}^{\infty}\frac{x^n}{(n-1)!}$

Differentiating both side, we have,

$e^x + xe^x = \sum_{n=1}^{\infty}\frac{nx^{n-1}}{\lfloor n-1}$

$= \sum_{n=1}^{\infty}\frac{n^2x^{n-1}}{n.\lfloor n-1}$

Multiplying on both side, we have,

$e^x + xe^x = \sum_{n=1}^{\infty}\frac{n^2}{\lfloor n}x^n$

Put $x = 2$, we have

$\sum_{n=1}^{\infty}\frac{n^2.2^n}{\lfloor n} = 2e^2 + 4e^2 = 6e^2$

Hence option (4) is correct

14. Explanation: We have

$u_n = \frac{(2n)!}{(n!)}$, then $u_{n+1} = \frac{(2(n+1))!}{((n+1)!)^2}$

$\therefore \frac{u_{n+1}}{u_n} = \frac{(2n+2)!}{(n+1)!}.\frac{(n!)^2}{(2n)!}$

$= \frac{(2n+2)(2n+1)}{(n+1)^2} = \frac{\left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)}{\left(1+\frac{1}{n}\right)}$

Therefore $\lim_{n\to\infty}\frac{u_{n+1}}{u} = 4 > 1$, he by D'Alembert ratio test, given inflite series diverge

Hence option (A) is

15. Explanation: Comer the infinite series $\sum v_n =$

$\sum\frac{(-1)^{n-1}}{n}\left(\sqrt{n+1} - \sqrt{n-1}\right)$

For absolute convergence, consider the infinite series $\sum|v_n|$

Then $|v_n| = \left|\frac{1}{n}\left(\sqrt{n+1} - \sqrt{n-1}\right)\right|$

$= \frac{1}{n}\frac{\left(\sqrt{n+1}-\sqrt{n-1}\right).\left(\sqrt{n+1}-\sqrt{n-1}\right)}{\left(\sqrt{n+1}-\sqrt{n-1}\right)}$

$= \frac{1}{n}\left(\frac{(n+1)-(n+1)}{\sqrt{n+1}-\sqrt{n-1}}\right)$

$= \frac{2}{n}\frac{1}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}\right)}$

Let, us consider $u_n = \frac{1}{n^{1/2}}$, then $\lim_{n\to\infty}\frac{v_n}{u_n} =$

$\lim_{n\to\infty}\frac{2}{\left(\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}\right)} = \frac{2}{2} = 1$

17. Explanation: Given that $S_n(x) = \sum_{r=1}^{n}\frac{x}{(rx+1)(r+1)x+1)}$

Note that, $\frac{x}{(rx+1)((r+1)x+1)} = \frac{1}{(rx+1)} - \frac{1}{(r+1)x+1}$ for all r

$\Rightarrow S_n(x) = \sum_{r=1}^{n}\left(\frac{1}{rx+1} - \frac{1}{(r+1)x+1}\right) = \frac{1}{x+1} - \frac{1}{(n+1)x+1}$

$= \frac{nx}{(x+1)((n+1)x+1)} = \frac{1}{(x+1)}\frac{nx}{(n+1)x+1}$

Therefore $\lim_{n\to\infty} S_n(x) = \frac{1}{x+1}.\lim_{n\to\infty}\frac{nx}{nx+x+1}$

$\frac{1}{x+1}.\lim_{n\to\infty}\frac{x}{x+\frac{x+1}{n}} = \frac{1}{x+1}$

$\Rightarrow f(x) = \lim_{n\to\infty} S_n(x) = \frac{1}{x+1}$

Clearly $f(x)$ is continuous for all x, except at one point $x = -1$.

Hence Option (4) is correct.

18. Explanation:

(A) $\sum_{n=0}^{\infty}(\sin x)^n = 1 + \sin x + \sin^2 x + \sin^3 x + \cdots =$

$4 + 2\sqrt{3}$

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As the given series is in G.P with first term $a = 1$ and common ratio

$r = \sin x$

$\therefore 1 + \sin x + \sin^2 x + \cdots = \frac{1}{1-\sin x}$

$4 + 2\sqrt{3} = \frac{1}{1-\sin x}$

$1 - \sin x = \frac{1}{4+2\sqrt{3}}$

$1 - \sin x = \frac{4-2\sqrt{3}}{(4+2\sqrt{3})(4-2\sqrt{3})}$

$= \frac{4-2\sqrt{3}}{16-12}$

$= \frac{4-2\sqrt{3}}{4}$

$= 1 - \frac{\sqrt{3}}{2}$

$\sin x = 1 - 1 + \frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

$\Rightarrow$ option (1) is correct

19. Explanation:

$\frac{1}{x+1} = 1 - x + x^2 - x^3 + \cdots.$

both true

20. Explanation:

(A) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$ is dgt

Option (B), (C) and (D) are direct result

$\Rightarrow$ option (A) is false

21. Explanation:

S_1 is true but S_2 is false.

22. Explanation:

(3) Let

$a_n = \left[(n+1)^{1/5} - n^{1/5}\right] = n^{1/5}\left[\left(1 + \frac{1}{n}\right)^{1/5} - 1\right]$

$= n^{1/5}\left[1 + \frac{1}{5}\left(\frac{1}{n}\right) + \frac{\left(\frac{1}{5}\right)\left(\frac{1}{5}-1\right)}{2!}\left(\frac{1}{n}\right)^2 + \frac{\left(\frac{1}{5}\right)\left(\frac{1}{5}-1\right)\left(\frac{1}{5}-2\right)}{3!}\left(\frac{1}{n}\right)^3 + \cdots - 1\right]$

$= \frac{1}{5}\left(\frac{1}{n^{4/5}}\right) + \frac{\left(\frac{1}{5}\right)\left(\frac{1}{5}-1\right)}{2!}\left(\frac{1}{n^{4/5}}\right) + \cdots$

Now, let $b_n = \frac{1}{n^{4/5}}$

$\therefore \lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{5} \neq 0$

As the given series is of the form of telescopic series

$\therefore a_1 = \left[(1+1)^{1/5} - 1\right] = \left[2^{1/5} - 1\right]$

$\therefore$ sum of series $= l = a_1$

$= \frac{1}{5} - 2^{1/5} + 1$

$= \frac{6}{5} - 2^{1/5}$

$= 1.2 - 1.14$

$= 0.06$

$\therefore$ The sum is greater then -1 but less then 2.

23. Explanation:

$\lim_{n\to\infty} \sum_{n=0}^{n} \frac{2k}{k^2+n^2} = \lim_{n\to\infty} \frac{1}{n} \sum_{n=0}^{n} \frac{2\left(\frac{k}{n}\right)}{\left(1+\frac{k^2}{n^2}\right)}$

$= \int_0^1 \frac{2x}{1+x^2} = [\log(1+x^2)]_0^1$

$= \log 2$

$\Rightarrow$ option (2) is correct

24. Explanation: (A),(B)

(1) By result

$\sum \frac{\log n}{n^p}$ is convergent for $p > 1$

(2) $a_n = \frac{n^2}{n!}$ And $a_{n+1} = \frac{(n+1)^2}{(n+1)!}$

$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(n+1)!} \times \frac{n!}{n^2}$

$= \left(\frac{(n+1)}{n}\right)^2 \times \frac{n!}{(n+1)n!}$

$= \frac{1}{n+1}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$

By ratio test the given series converges

(3) $\sum \frac{1}{n(\log n)^p}$ is convergent $\Leftrightarrow p > 1$ but here $p = 1$

$\Rightarrow$ The series is not convergent

(4) Let $a_n = \frac{e^n}{n^{100}}$ and $a_{n+1} = \frac{e^{n+1}}{(n+1)^{100}}$

$\therefore \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{e^{n+1}}{(n+1)^{100}} \times \frac{n^{100}}{e^n}$

$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{100} \times e^{n+1-n}$

$= e > 1$

$\Rightarrow$ Divergent

$\Rightarrow$ Option (1) and (2) are correct

25. Explanation: Let $a_n = \frac{1}{\sqrt{n}}$ and $a_{n+1} = \frac{1}{\sqrt{n+1}}$

$\therefore 1/R = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\sqrt{n}}{\sqrt{n+1}}\right|$

$\Rightarrow R = 1$

$\therefore$ interval is $|x| \leq 1$

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$\Rightarrow -1 \le x \le 1$

$\Rightarrow$ option (1) is correct

26. Explanation:

This is a telescopic series and $\sum b_n$ is convergent iff

(b_n) is convergent iff

(a_n) is convergent iff

$\Rightarrow$ option (1) is correct

27. Explanation: Given: (a_n) and (b_n) are real sequence

and $\sum a_n b_n$ converges

$\Rightarrow |a_n| < m, \forall m \in \mathbb{N}$

$\Rightarrow |a_n . b_n| \le m . b_n$

$\sum b_n$ converges

$\Rightarrow \sum m . b_n$ is convergent

$\Rightarrow \sum a_n b_n$ is convergent

$\Rightarrow$ If $\sum b_n$ converges then (a_n) is bounded

$\Rightarrow$ option (4) is correct

28. Explanation:

$a_n = \log \cos\left(\frac{1}{n}\right)$

By taking n^{th} root

$(a_n)^{1/n} = \log\left(\cos\left(\frac{1}{n}\right)^{1/n}\right)$

$= \frac{1}{n}\log\left(\cos\left(\frac{1}{n}\right)\right)$

$\therefore$ By cauchy's root test

$\lim_{n\to\infty}(a_n)^{1/n} = \lim_{n\to\infty}\frac{1}{n}\log\left(\cos\left(\frac{1}{n}\right)\right)$

$\lim_{n\to\infty}\frac{1}{n}\lim_{n\to\infty}\log\left(\cos\left(\frac{1}{n}\right)\right)$

$0 \times 0 = 0 < 1$

$\therefore$ by Cauchy's root test

The given series converges absolutely

29. Explanation:

$S_1 : a_n = \left(1 - \frac{1}{2^n}\right)^n$

$\lim_{n\to\infty}\left(1 - \frac{1}{2^n}\right)^n = e^{\lim_{n\to\infty} n\left(1-\frac{1}{2^n}-1\right)}$

$= e^{\lim_{n\to\infty} -\frac{n}{2^n}}$

$= e^0 = 1 \neq 0$

$\Rightarrow$ not convergent

$S_2 : a_n = \frac{\log^3 n}{n}$

Let, $a(x) = \frac{\log^3(x)}{x}$ and

$a'(x) = \frac{x\left[3\log^2(x).\,1/x\right] - \log^3(x)}{x^2}$

$= \frac{3\log^2(x) - \log^3(x)}{x^2}$

As $x > 1000$

$= \log x > 3$

$= \log^3(x) > 3\log^2(x)$

$= 3\log^2(x) - \log^3(x) < 0$

$= \lim_{n\to\infty} a_n = 0$

$\therefore$ by leibnitz test the given series is cgt

30. Explanation:

A) Given $|a_n| \le \frac{n+1}{n^3}, \forall\, n$

And $\sum_{n=1}^{\infty}\frac{n+1}{n^3}$ is cgt :

As $S_n = \frac{n+1}{n^3}$ and take $t_n = \frac{1}{n^2}$

$\lim_{n\to\infty}\frac{S_n}{t_n} = \lim_{n\to\infty}\frac{n + 1/n^3}{1/n^2} = 1 \neq 0$

But $\sum t_n = \sum 1/n^2$ is cgt

$\therefore \sum\frac{n+1}{n^3}$ is cgt

i.e. $|a_n| \le |b_n|, \forall\, n$ and $\sum b_n$ is cgt

$\therefore \sum|a_n|$ is cgt absolutely

$\Rightarrow$ option (A) and (B) are wrong

D) Let $0 \le A_n \le \frac{n^2+1}{n^3}$

Let $= \frac{n^2}{n^3} \le a_n$

But $\sum b_n = \sum\frac{1}{n}$ is dgt series

$\therefore \sum a_n$ is not cgt.

$\Rightarrow$ option (D) is wrong

31. Explanation:

Let $\frac{1}{1001} + \frac{1}{1002} + \cdots + \frac{1}{2000}$

$= 1000\left(\frac{1}{1001}\right) < 1$

And the sum is more than 0 and $\log 2 = 0.693$

$\therefore$ The sum is less than log 2 but not more than log 2

32. Explanation:

1) When the radius of convergence of a sequence is R and the $\sum a_n$ converges to R then the $\sum a_n$ converges on $(-R, R)$ and not on $|x| = R$

$\Rightarrow$ statement (A) is incorrect

3) lly for divergent

When radius of convergence of a sequence is R. and $\sum a_n$ diverges to R then $\sum a_n$ diverges on $(-R, R)$ and not on $|x| = R$

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⇒ statement (3) is incorrect

⇒ statement (2) and (4) are correct

33. Explanation:

Both A and B are false

34. Explanation: (2)

$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is convergent $for\ P > 1$

So (1) is incorrect (2) correct

(3) $\sum_{n=1}^{\infty} \cos n$

$\lim_{n\to\infty} \cos n \neq 0$

$\Rightarrow \sum_{n=1}^{\infty} \cos n$ diverges

(4) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$

$\lim_{n\to\infty} \cos\left(\frac{1}{n^2}\right) = 1 \neq 0$

$\Rightarrow \sum \cos\left(\frac{1}{n^2}\right)$ diverges

35. Explanation:

$\sum_{k=1}^{\infty} \frac{k^2}{k!} = \sum_{k=1}^{\infty} \frac{k}{(k-1)!} = \sum_{k=1}^{\infty} \frac{k+1-1}{(k-1)!}$

$= \sum_{k=1}^{\infty} \frac{k-1}{(k-1)!} + \sum_{k=1}^{\infty} \frac{1}{(k-1)!}$

$= \sum_{k=1}^{\infty} \frac{1}{(k-2)!} + \sum_{k=1}^{\infty} \frac{1}{(k-1)!}$

$= e + e = 2e$

36. Explanation:

$\sum_{k=1}^{\infty} \sqrt{\log k}\ x^k$

$a_k = \sqrt{109k}$

$\frac{1}{p} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\log(k+1)^{\frac{1}{2}}}{\log(k)^{\frac{1}{2}}}$

$\left(\frac{\log(k+1)}{\log(k)}\right)^{\frac{1}{2}}$

$\frac{1}{R} = 1$

$R = 1$

37. Explanation:

$\sum_{k=1}^{\infty} \frac{(-1)^n}{n^x}$.

This is alternating series by leibnit'z test

This is convergent $for\ \propto \geq 1$

38. Explanation:

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k^2}}$

$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\sqrt{1+\left(\frac{k}{n}\right)^2}} = \int_0^1 f(x)dx$

$f(x) = \frac{1}{\sqrt{1+x^2}}$

$\int_0^1 \frac{1}{\sqrt{1+x^2}} dx = \left[\ln\left|\sqrt{x^2+16}+x\right|\right]_0^1$

$= \ln\left|\sqrt{2}+1\right|$

39. Explanation:

$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n} \cos\left(\frac{\pi k}{2n}\right) = \int_0^1 \cos\left(\frac{\pi x}{2}\right) dx$

$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n} \cos\left(\frac{\pi k}{2n}\right) = \left[\sin\left(\frac{\pi x}{2}\right)\left(\frac{2}{\pi}\right)\right]_0^1$

$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n} \cos\left(\frac{\pi k}{2n}\right) = \frac{2}{\pi}$

40. Explanation:

$(x+1) - \frac{(x+1)^3}{9} - \frac{(x+1)^4}{10}$

$\sum \frac{(x+1)^n}{n^2} = \sum \frac{(x+1)^n}{n^2}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(x+1)^{n+1}}{(n+1)^2} \times \frac{n^2}{(x+1)^n}$

$= \lim_{n\to\infty} (x+1) \times \frac{n^2}{(n+1)^n}$

$= (x+1)$

This limit exists if

$|x+1| \leq 1$

$-1 \leq x+1 \leq 1$

$-2 \leq x \leq 0$

41. Explanation: We have

$1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)n+1}{n^p}$

This an alternating series by Leibnitz series test series converges for P>0

42. Explanation:

1. $\sum_{n=1}^{\infty} \sqrt{\frac{2n^2+3}{5n^3+7}}$

$a_n = \sqrt{\frac{2n^2+3}{3n^3+7}}$

Let $b_n = \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{\frac{2n^2+3}{3n^3+7}}}{\frac{1}{\sqrt{n}}} = \sqrt{\frac{2}{3}} \neq 0$

Therefore $\sum b_n$ divergent so $\sum_{n=1}^{\infty} \sqrt{\frac{2n^2+3}{5n^3+7}}$ divergent

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2. $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+\frac{3}{2}}}$

$a_n = \frac{(n+1)^n}{n^n + \frac{3}{2}} \leq \frac{1}{n^{\frac{3}{2}}}$

$\sum \frac{1}{n^{\frac{3}{2}}}$ is convergent

$\Rightarrow \sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+\frac{3}{2}}}$ is convergent

3. $\sin\left(\frac{1}{n}\right) \leq \frac{1}{n^2}$

By direct comparison Test

$\sum \frac{1}{n} \sin\left(\frac{1}{n}\right)$ is convergent

43. Explanation: $\sum_{n=1}^{\infty} \frac{\log(n+1)}{\sqrt{n+1}} (x-5)^n$

$a_n = \frac{\log(n+1)}{\sqrt{n+1}} (x-5)^n$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\log(n+2)}{\sqrt{n+2}} \frac{(x-5)^{n+1}}{(x-5)^n} \times \frac{\sqrt{n+j}}{\log(n+j)}$

$\lim_{n\to\infty} \sqrt{\frac{n+1}{n+2}} \frac{\log(n+2)}{\log(n+1)} \times (x-5)$

$\lim_{n\to\infty} \frac{\log(n+2)}{\sqrt{n+2}} = \lim_{n\to\infty} \frac{n+1}{n+2} = 1$ (by L hospital)

$\lim_{n\to\infty} \frac{a_n + 1}{a_n} = (x-5)$

The Series is convergent

$|x-5| \leq 1$

$-< x-5 < 1 \Rightarrow 4 \leq x \leq 6$

45. Explanation:

1. $\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}, \quad x \in IR$

$|a_n| = \frac{\cos n_1}{n^2} \leq \frac{1}{n^2}$

$\Rightarrow \sum |a_n|$ – converges

$\Rightarrow \sum a_n$ is converges absolutely

2. $a_n = (-1)^n \frac{n}{n+2}$

$|a_n| = \frac{n}{n+2}$

$\lim_{n\to\infty} an = 1$

$\sum |a_n|$ is not convergent

$\Rightarrow \sum a_n$ is divergent

3. $a_n = (-1)^{n-1} \frac{1}{2n+3}$

$\in n^{n-1}$ is bounded

$4 \frac{1}{2n+3} \to 0$ & monotonically decreasing by Leibnitz test

$\sum a_n = \sum \frac{(-1)^{n-1}}{2n+3}$ converges

But $\sum |a_n| = \sum \frac{1}{2n+3}$ is not convergent

$\Rightarrow \sum a_n$ conditionally convergent

46. Explanation:

$\sum \frac{(-1)^n}{\sqrt{n}} = \sum a_n = \sum b_n$

$\sum a_n b_n = \sum \frac{1}{n}$

Here

$\sum a_n$ & $\sum b_n$ convergent

$\sum a_n b_n$ is not convergent

c) correct

If the series $\sum_n a_n$ is convergent, $a_n \geq 0$ for all n, and if the sequence $\{b_n\}$ is bounded, then $\sum_n a_n b_n$ is absolutely convergent

47. Explanation:

1. $\sum_{n=1}^{\infty} \left(n^{\frac{1}{n}} - 1\right)^n$

Here $a_n = \left(n^{\frac{1}{n}} - 1\right)^n$

By root test

$\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} (n^{\frac{1}{n}} - 1)$

$= 0 < 1$

$\Rightarrow$ By Cauchy root test

$\sum_{n=1}^{\infty} a_n$ is convergent

2. $\sum a_n$ is convergent

$\frac{\sqrt{a_n}}{n} = \frac{\sqrt{a_n}}{n} = \sqrt{\frac{1}{n^2} a_n}$

$\sum \sqrt{a_n} \frac{1}{n^2}$ is convergent

48. Explanation:

1. $\sum |a_n|$ is divergent

$\Rightarrow \sum a_n$ does not convergent absolutely

2. $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$

$|a_n| = \frac{1}{n} \sin \frac{1}{n}$

$\sin\left(\frac{1}{n}\right) < \frac{1}{n}$

$\frac{1}{n} \sin\left(\frac{1}{n}\right) < \frac{1}{n^2}$

$\sum |a_n|$ is cgt

$\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \sin\left(\frac{1}{n}\right)$ is absolutely cgt

3. $\sum (-1)^n \frac{n}{(n+1)(n+2)}$

$|a_n| = \frac{n}{(n+1)(n+2)} < \frac{1}{n^2}$

$\sum |a_n|$ is convergent

$\Rightarrow \sum a_n$ is absolutely

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49. Explanation:

$$S = \frac{1}{6} + \frac{5}{6.12} + \frac{5.8}{6.12.18} + \frac{5.8.11}{6.12.18.24} + \cdots$$

$$2S = \frac{2}{6} \times \frac{2}{2} + \frac{2.5}{6.12} \times \frac{2^2}{2^2} + \frac{2.5.8}{6.12.18} + \frac{2.5.8.11}{6.12.18.24}$$

$$= \frac{2}{6} \times \frac{2}{2} + \frac{2.5}{6.12}\frac{2^3}{2^3} + \cdots \frac{+2.5.8.11}{6.12.18.24}\frac{2^3}{2^3} + \cdots$$

$$= 1 + \left(\frac{-1}{2}\right)\left(\frac{-2}{3}\right) + \frac{\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)}{2!}\left(\frac{-1}{2}\right)^2$$

$$(1-x)^n = 1 - nx + \frac{n(n+1)}{2!}2 + \cdots$$

$$\left(1-\frac{1}{2}\right)^{\frac{-2}{3}} = 1 + \left(\frac{-1}{2}\right)\left(\frac{-2}{3}\right) + \frac{\left(\frac{-2}{3}\right)\left(\frac{-2}{3}-1\right)}{2!}\left(\frac{-1}{2}\right) + ..$$

$$\left(1-\frac{1}{2}\right)^{\frac{-2}{3}} = \left(\frac{1}{2}\right)^{-\frac{2}{3}}$$

$$2S = \left(\frac{1}{2}\right)^{-\frac{2}{3}}$$

$$S = 1$$

50. Explanation:

1. $\sum_{n=1}^{\infty}\left(3\sqrt{n^3+1} - n\right)$

$$(n^3+1)^{\frac{1}{3}} - n = \frac{n^3+1-n^3}{(n^3+1)^{2/3} + n(n^3+1)^{1/3} + n^2}$$

$$= \frac{1}{(n^3+1)^{2/3} + n(n^3+1)^{1/3} + n^2}$$

$$\leq \frac{1}{n^2}$$

$\Rightarrow$ by direct comparison Test

$\sum_{n=1}^{\infty} 3\sqrt{n^3+1} - n$ is convergent

2. $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \cdots$

$$\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)(n+1)(n+3)} \leq \frac{1}{n^2}$$

$\Rightarrow$ by direct comparison

$\sum an$ is convergent

We can easily see $f(x)$ is continuous

51. Explanation:

1. $\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+\frac{3}{2}}}$

$$a_n = \left(\frac{n+1}{n}\right)\frac{1}{n^{3/2}}$$

Let $b_n = \frac{1}{n^{3/2}}$

$$= \left(\frac{n+1}{n}\right)^n$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \neq 0$$

$\Rightarrow \sum b_n$ convergent

$\Rightarrow \sum a_n$ convergent

2. $\sum \frac{1}{n^{1+\frac{1}{n}}}$

$$a_n = \frac{1}{n^{1+\frac{1}{n}}}$$

Let $b_n = \frac{1}{n}$

$$\lim_{n\to\infty}\frac{an}{bn} = \lim_{n\to\infty}\frac{\frac{1}{n^{1+\frac{1}{n}}}}{1/n} = \lim_{n\to\infty}\frac{1}{n^{1/n}} = 1 \neq 0$$

$\sum \frac{1}{n}$ is divergent

$\sum a_n$ is divergent

52. Explanation:

$$1 + \frac{(\log a)^2}{2!} + \frac{(\log a)^4}{4!} +$$

$$\sum_{n=0}^{\infty}\frac{\pm 2n}{2n!} = \cosh t$$

Put $t = \log a$

$$\sum_{n=0}^{\infty}\frac{(\log a)^{2n}}{2n!} = \cosh(\log a)$$

$$\cosh(\log a) = \frac{e^{\log a} + e^{-\log a}}{2}$$

$$\cosh(\log a) = \frac{1}{2}\left(a + \frac{1}{a}\right)$$

53. Explanation:

(1) $\sum_{n=1}^{\infty}\sqrt{\frac{2n^2+3}{5n^3+1}}$

$$a_n = \sqrt{\frac{2n^2+3}{5n^3+1}}$$

$$\frac{n}{n^{3/2}}\sqrt{\frac{2+3/n^2}{5+1/n^3}}$$

$$a_n = \frac{1}{\sqrt{n}}\sqrt{\frac{2+3/n^2}{5+1/n^3}}$$

Let $b_n = \frac{1}{\sqrt{n}}$

$$\frac{a_n}{b_n} = \sqrt{\frac{2+3/n^2}{5+1/n^3}}$$

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \sqrt{\frac{2}{5}} \neq 1$$

$\sum \frac{1}{\sqrt{n}}$ is divergent

$\Rightarrow \sum_{n=1}^{\infty}\sqrt{\frac{2n^2+3}{5n^3+1}}$ is divergent

(2) $\sum_{n=1}^{\infty} n^2x(1-x^2)^n$

$$a_n = n^2\,x(1-x^2)^n$$

$$\lim_{n\to\infty}\frac{a_n+1}{a_n} = \frac{(n+1)^2x(1-x^2)^{n+1}}{n^2x(1-x^2)^n}$$

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)(1-x^2)$$

$$\lim\left(1 - \frac{1}{}\right)(1-x^2)$$

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$$\lim_{n\to\infty}\frac{a_n+1}{a_n}=1-x^2<0(\therefore 0<x<1)$$

$\Rightarrow$ by ratio Test

$\sum n^2x(1-x^2)^n$ is converge

54. Explanation:

(1) $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+\ldots$

$\sum_{n=0}^{\infty}\frac{(-1)^n}{4n+1}$

Here by Leibnitz test we can say $\sum_{n=0}^{\infty}\left|\frac{(-1)^n}{4n+1}\right|$ is not convergent

$\Rightarrow \sum_{n=0}^{\infty}\frac{(-)^n}{4n+1}$ is conditionally convergent

(2) $\sum_{n=1}^{\infty}\frac{(-1)^n n}{n+1}$

$\lim_{n\to\infty}(-1)^n\to 1$

$\lim_{n\to}(-1)^n\to -1$

$\lim_{n\to\infty}\frac{n}{n+1}=1$

$\Rightarrow \lim_{n\to\infty}\frac{n^{(-1)}x}{n+1}\neq 0$

$\Rightarrow \sum_{n=0}^{\infty}\frac{(-1)^x n}{n+1}$ is divergent

(3) $\sum_{n=2}^{\infty}\frac{(-1)^x}{n(n-1)}$

By elbnitz test

$\sum_{n=2}^{\infty}\frac{(a-1)^x}{n(n-1)}$ is convergent

$\sum_{n=2}^{\infty}\left|\frac{(-1^n)}{n(n-1)}\right|=\sum_{n=2}^{\infty}\frac{1}{n(n-1)}$

$\frac{1}{n(n-1)}<\frac{1}{n^2}$

$\Rightarrow \sum\frac{1}{n(n-1)}$ is convergent

$\Rightarrow \sum\frac{(-1)^n}{n(n-1)}$ is absolutely convergent

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[CUET PG]

[CUET PG 2022]

1. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Let Q be the set of rational numbers

Assertion A:

Q is not complete.

Reason R:

The set $\{x \in Q : x^2 < 2\}$ has no supremum in Q.

In the light of the above statements, choose the correct answer from the options given below

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is not the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

[CUET PG 2023]

2. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\left(\frac{f(1+x)}{f(1)}\right)^{\frac{1}{x}}$ equals

(1) $e^{\frac{1}{2}}$ (2) e^2

(3) e^3 (4) 1

Answer Key	
1	2
1	2

[CU-CET]

[CUCET 2016]

1. The $\frac{loglog\,(coscos\,x)}{x}$

(1) does not exist

(2) exists and its value is $\frac{-1}{2}$

(3) exists and its value is 0

(4) exists and its value is $\frac{1}{2}$

[CUCET 2016]

2. Consider the function $f(x) = \frac{1}{1+e^{\frac{1}{x}}}$ for $x \neq 0$. Then

(1) Left hand limit $f(x)$ at $x = 0$ exists but the right hand limit $f(x)$ at $x = 0$ does not exist

(2) $f(x)$ at $x = 0$ exists but $f(x)$ does not exist

(3) Both $f(x)$ and $f(x)$ at $x = 0$ exist and they are equal

(4) Both $f(x)$ and $f(x)$ at $x = 0$ exist and they are not equal

[CUCET 2016]

3. The value of $x\left(log\left(1+\frac{x}{2}\right) - log\left(\frac{x}{2}\right)\right)$ is

(1) 2 (2) 1

(3) 0 (4) -1

[CUCET 2017]

4. A monotonic function

(1) is always continuous

(2) is continuous only, if it satisfies intermediate value property

(3) can be nowhere continuous

(4) can be discontinuous at infinitely many points

[CUCET 2019]

5. Every compact set of real numbers is

(1) open

(2) closed

(3) closed and bounded

(4) open and bounded

[CUCET 2019]

6. The set R of real real numbers is

(1) closed (2) bounded

(3) countable (4) none of the above

[CUCET 2020]

7. $\frac{x-|x|}{x}$ equals

(1) 0 (2) 1

(3) 2 (4) limit does not exist

Answer Key						
1	2	3	4	5	6	7
2	4	1	2	3	1	4

[PRACTICE-QUESTIONS]

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function with $f(0) = f(1) = f'(0) = 0$. Then

(1) f" is the zero function.

(2) $f''(0)$ is zero.

(3) $f(x) = 0$ for some $x \in (0,1)$.

(4) f'' never vanishes.

2. Which of the following statements is not correct for a real valued function f?

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(1) If f is Riemann integrable on $[a,b]$, then f^2 is also Riemann integrable on $[a,b]$.

(2) If f^2 is Riemann integrable on $[a,b]$, then f is also Riemann integrable on $[a,b]$.

(3) If f^3 is Riemann integrable on $[a,b]$, then f is also Riemann integrable on $[a,b]$.

(4) If f is Riemann integrable on $[a,b]$, then $|f|$ is also Riemann integrable on $[a,b]$.

3. If $f(x)=\sqrt{x}, \phi(x)=\frac{1}{\sqrt{x}}$ are defined on the interval [1, 2], then the value of C satisfying Cauchy's mean value theorem is

(1) $\sqrt{3}$ (2) $\sqrt{2}$

(3) $2+\sqrt{2}$ (4) $1+\sqrt{2}$

4. If $f(x)=x[x]$, where $[x]$ denotes the greatest integer not greater than x, then

(1) f is integrable on [0,2] and $\int_0^2 f\,dx=\frac{3}{2}$.

(2) f is integrable on [0,2] and $\int_0^2 f\,dx=\frac{2}{3}$.

(3) f is integrable on [0,2] and $\int_0^2 f\,dx=0$.

(4) f is not integrable on [0,2].

5. If, for $x,y,\in\mathbb{R}\ x\sim y \Leftrightarrow 3+4x=2y$ then:

(1) $1-\frac{7}{2}$ (2) $2-\frac{1}{7}$

(3) $-1-\frac{7}{2}$ (4) $-2-\frac{1}{7}$

6. Which of the following statements is false for real valued functions?

(1) Every bounded function defined on [a, b] having finite number of points of discontinuity is Riemann integerable over [a, b].

(2) Every function for which Riemann integeral exists is continuous on [a, b].

(3) Every monotonic function defined on [a, b] is Riemann integerable over [a, b].

(4) Every continuous function defined on [a, b] is Riemann integerable over [a, b].

7. A function $f:[-1,1]\to\mathbb{R}$ is defined as $f(x)=\begin{cases}2, & x\neq 0\\ 0, & x=0\end{cases}$ then:

(1) f is continuous at $x=0$.

(2) f is a monotonic increasing function.

(3) f is not Riemann integrable over $[-1,1]$.

(4) f is Riemann integrable over $[-1,1]$.

8. If $f(4)=4$ and $f'(4)=1$, then value of $\lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}$ is:

(1) 0 (2) 1

(3) -1 (4) 4

9. Let f be a real-valued differentiable function. Let $h(x)=\frac{1}{3}\{f(x)\}\&3+\{f(x)\}^2+f(x)+\frac{1}{3}, x\in\mathbb{R}$. Which of the following statements is true?

(1) f is monotonic increasing his monotonic increasing.

(2) f is mx,yonotonic decreasing his monotonic increasing.

(3) h is monotonic increasing, whether f is monotonic Increasing or monotonic decreasing.

(4) h is monotonic decreasing, whether f is monotonic increasing or monotonic decreasing

10. The maximum value of $\left(\frac{1}{x}\right)^x, x>0$ is :

(1) v^u (2) $1/e$

(3) $e^{\frac{1}{e}}$ (4) 0

11. For a function $f(x,y)=x^3y, x,y\in\mathbb{R}, \frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ are equal :

(1) Only at X-axis (2) Only at Y-axis

(3) at X-axis and at Y-axis (4) always

12. If $f(x,y)$ is a homogeneous function of degree p and f is differentiable, then $x\frac{\partial}{\partial x}f(x,y)+y\frac{\partial}{\partial x}f(x,y)=$

(1) $p, f(x,y)$ (2) $p(q-1)f(x,y)$

(3) $p\frac{\partial}{\partial x}f(x,y)$ (4) $p\frac{\partial}{\partial y}f(x,y)$

13. If $f:X\to Y$ and $g:Y\to Z$ are maps, then the incorrect statement is:

(1) f and g are one-one onto $\Rightarrow g$ of is one-one onto

(2) g of is one-one onto $\Rightarrow f$ is one-one

(3) g of is one-one onto $\Rightarrow g$ is onto

(4) g of is one-one onto $\Rightarrow f$ is onto

14. If x is a real number such that $|1-x|=x-1$ then :

(1) $x<1$ (2) $x>1$

(3) $x\geq 1$ (4) $x\leq 1$

15. If f is a real valued function defined as $f(x)=x^{3/2}$, then:

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(1) domain $(f) = (-\infty, \infty)$ range (f)
(2) domain $(f) = (-\infty, \infty)$ and range $(f) = (0, \infty)$
(3) domain $(f) = (0, \infty)$ and range $(f) = (-\infty, \infty)$
(4) domain $(f) = (0, \infty)$ and range $(f) = (0, \infty)$

16. If f and g are real valued functions defined as $f(x) = 2|x| + 1$ and $g(x) = \frac{1}{x^2-1}$ then :
(1) f is an even function and g is odd function
(2) g is an even function and f is odd function
(3) both f and g are even functions
(4) both f and g are odd functions

17. The real valued function $f(x) = 3 - 2x - x^2$ represents a :
(1) Parabola (2) Ellipse
(3) Circle (4) Hyperbola

18. Let $f(x) = x^3 - x - 1$ be a real polynomial. Then:
(1) f has a zero in the interval [0, 1]
(2) f has a zero in the interval [1, 2]
(3) f has a zero in the interval [2, 3]
(4) f has no real zero

19. Let $f : IR \to IR$ be a linear function such that $f(1) = \alpha$, then
(1) f is continuous but not differentiable
(2) f is continuous and differentiable and $f'(x) = \alpha$ for all $x \in IR$
(3) f is neither continuous nor differentiable
(4) $f'(x) = 0$ for all $x \in IR$

20. Which of the following statements in not true?
(1) Every continuous function is Riemman integrable
(2) Every differentiable function is Riemman integrable
(3) Every bounded function is Riemman integrable
(4) None of the above

21. A function $f : |0,3| \to \mathbb{R}$ is defined by $f(x) = |x| + |x-1| + |x-2| + |x+3|, \forall x \in [0,3]$ The number of points in $[0,3]$, where f is not differentiable, is
(1) 3 (2) 2
(3) 4 (4) 1

22. If $f : [a,b] \to \mathbb{R}$ and $g : [a,b] \to \mathbb{R}$ are two functions such that $f'(x) = g'(x), \forall x \in [a,b]$, where f' and g' denote first derivate of f and g respectively, then $(f-g)(x), \forall \in [a,b], c$ being any real constant, is
(1) c (2) cx
(3) cx^2 (4) cx^3

23. If a function $f : \mathbb{R} \to \mathbb{R}$ is twice derivable and satisfies $\forall x > a > 0$, the inequalities $|f(x)| < A, |f'(x)| < B$, where A and B are constants and $f'(x), f'(x)$ denote first and second derivative of $f(x)$ respectively, then $\forall x > a > 0, |f'(x)|$ is less than
(1) $2\sqrt{AB}$ (2) $\sqrt{AB}$
(3) $\frac{1}{3}\sqrt{AB}$ (4) $\frac{1}{4}\sqrt{AB}$

24. If a function $f : [0,1] \to \mathbb{R}$ is defined by $f(x) = 2rx$, when $\frac{1}{r+1} < x < \frac{1}{r}$ and r is a positive integer, then $\int_0^1 f(x)\,dx$ is
(1) $\frac{\pi}{6}$ (2) $\frac{\pi^2}{6}$
(3) $\frac{\pi}{3}$ (4) $\frac{\pi^2}{3}$

25. $\int_0^1 x^m(1-x)^n\,dx$ exists if
(1) $m > -1, n < -1$ (2) $m < -1, n < -1$
(3) $m > -1, n > -1$ (4) $m < -1, n > -1$

26. If $y = \left[\log\left(x + \sqrt{(1+x^2)}\right)\right]^2$ and n is a positive integer, then $(y_{n+2})_{(0)}$ is
(1) $-n(y_n)_{(0)}$ (2) $n(y_n)_{(0)}$
(3) $n^2(y_n)_{(0)}$ (4) $-n^2(y_n)_{(0)}$

27. With the help of the mean value theorem, if $0 < \theta < 1$ then the value of $\log_{10}(x+1)$ is
(1) $\frac{1}{1+\theta x}$ (2) $\frac{x^2}{1+\theta x}$
(3) $\frac{x \log_{10} e}{1+\theta x}$ (4) $\frac{x}{1+\theta x}$

28. A real root of the equation $x^3 - 3x - 5 = 0$ lies in
(1) (0,1) (2) (1,2)
(3) (2,3) (4) (3,4)

29. If $f(x) = \begin{cases} 1, when\ x\ is\ rational \\ -1, when\ x\ is\ irrational \end{cases}$ then $\int_a^b |f(x)|dx$ equal to
(1) $-(b-a)$ (2) $(b-a)$
(3) [illegible] (4) $\frac{b-a}{2}$

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30. A function f is defined in $[0,1]$ as follows:
$f(x) = p/q$, when x is any non-zero rational number p/q in its lowest terms and $f(x) = 0$, when x is irrational or 0.
Then the Riemann integral of f in $[0, 1]$ is
(1) 0 (2) 1
(3) -1 (4) $\frac{1}{2}$

31. The minimum value of $x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$ is attained at
(1) $(2,2)$ (2) $(1,1)$
(3) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{3}{2}, \frac{3}{2}\right)$

32. The function $x^3 + y^3 - 3axy$ has a maximum or minimum at the point _____.
(1) (a, a) (2) $(0,0)$
(3) $(a, 0)$ (4) $(0, a)$

33. The coefficient of x^4 in the Maclaurin's expansion of $\log \cos x$ is
(1) $-\frac{1}{24}$ (2) $-\frac{1}{12}$
(3) $-\frac{1}{45}$ (4) $-\frac{1}{6}$

34. If we expand $\cos\left(\frac{\pi}{4} + \theta\right)$ in powers of θ, the coefficient of $\frac{\theta^3}{3!}$ is
(1) $\frac{1}{\sqrt{2}}$ (2) $-\frac{1}{\sqrt{2}}$
(3) $\frac{1}{2}$ (4) None of the above

35. Let $f: A \to A$ and $B \subset A$. Then which one of the following is always true?
(1) $B \subset f^{-1}(f(B))$ (2) $B = f^{-1}(f(B))$
(3) $B \subset f(f^{-1}(B))$ (4) $B = f(f^{-1}(B))$

36. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x^3$, then f is
(1) uniformly continuous
(2) not differentiable
(3) continuous but not uniformly continuous
(4) not continuous.

37. Let f be a real-valued function on $[0,1]$ such that $f(0) = -1$ and $f(1)$ 1/2, then there always exists a $t \in (0,1)$ such that
(1) $f'(t) = -2$ (2) $f'(t) = 1$
(3) $f'(t) = 3/2$ (4) $f'(t) = -1/2$

38. The number of solutions of the equation $3^x + 4^x 5^x$ in the set of positive real numbers is exactly
(1) 1 (2) 2
(3) 3 (4) 5

39. Let $f: R \to \mathbb{R}$ be differentiable and f' be bounded. Then
(1) f has a local maximum at exactly one point of $\mathbb{R}$
(2) f has a local maximum at exactly two point of $\mathbb{R}$
(3) f is uniformly continuous on $\mathbb{R}$
(4) $f + f'$ is uniformly continuous on $\mathbb{R}$

40. Let $f: [0, \pi/2] \to \mathbb{R}$ be continuous and satisfy $(\int_0^{\sin x} f(t)dt = \frac{\sqrt{3}x}{2} for\ 0 \le x \le \pi/2$ Then $f(1/2)$ equals
(1) ½ (2) $1/\sqrt{2}$
(3) 1/3 (4) 1

41. The function $f: [0,1] \to [0,1]$ defined by $f(0) = 0$ and $f(x) = x^2 \sin(1/x)\ for\ x \neq 0$, is not true
(1) differentiable on (0,1)
(2) is continuous on (0, 1)
(3) is continuous on [0, 1] but not differentiable at 0
(4) is uniformly continuous.

42. Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \min\{(x, x+1, x-2]\}$. Then which of the following is true?
(1) f is continuous on $\mathbb{R}$
(2) f is not differentiable at exactly two points
(3) f increases on the interval $(-\infty, 1]$
(4) f decreases on the interval [1,2] .

43. Let $f: I \to \mathbb{R}$ be an increasing function where I is an interval in $\mathbb{R}$. Then
(1) f^2 is always increasing
(2) f^2 is always decreasing
(3) f^2 is constant $\Rightarrow$ is constant
(4) f^2 may be neither decreasing nor increasing.

44. Let $f: [0,1] \to \mathbb{R}$ be a continuous function such that $f(x^2) = f(x)$ for all $x \in [0,1]$. Which one of the following not true in general?
(1) f is constant
(2) f is uniformly continuous
(3) f is differentiable
(4) $f(x) \ge 0 \forall x \in [0,1]$

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45. Let $f:[0,1] \to [0,1]$ be a continous function and $I:[0,1] \to [0,1]$ be the identity function. Then f and I
(1) agree exactly at one point
(2) agree at least at one point
(3) may not agree at any point
(4) agree at most at one point.

46. For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer n such that $n \le x$. The function $h(x) = x[x]$ is
(1) continuous everywhere
(2) continuous only at $x = \pm 1, \pm 2, \pm 3$
(3) continuous if $x \neq \pm 1, \pm 2, \pm 3$
(4) bounded on $\mathbb{R}$.

47. Consider the function $g: \mathbb{R} \to \mathbb{R}$ defined by

$$g(x) = \begin{cases} 0, & if\ x \ge 0, \\ e^{-1/x^2}, & if\ x \ge 0, \end{cases}$$

Which one of the following is not true?
(1) g has derivatives of all orders at every point
(2) $g^n(0) = 0$ for all $n \in \mathbb{N}$
(3) Taylor Series expansion of g about $x = 0$ converges tog for all x
(4) Taylor Series expansion of g about $x = 0$ converges to g for all $x \ge 0$

48. The faction $f(x) = x \sin x + \frac{1}{1+x^2}$; $x \in I$ where $I \subseteq \mathbb{R}$ is
(1) uniformly continuous if $I = \mathbb{R}$
(2) uniformly continuous if I is compact
(3) uniformly continuous if is closed.
(4) not uniformly continuous on [0,1].

49. Let the function $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x^2, & if\ x \in (0,2) \cap \mathbb{Q} \\ 2x - 1, & if\ x \in (0,2) \cap (\mathbb{R} \setminus \mathbb{Q}) \end{cases}$$

Which one of the following is not true?
(1) f is continuous at $x = 1$
(2) f is differentiable at $x = 1$
(3) f is not differentiable at $x = 1$
(4) is differentiable only at $x = 1$.

50. Let f be a function on $[0, \infty)$ by $f(X) = [x]$, the greatest integer less than or equal to x. Then
(1) f is continuous at each point of $\mathbb{N}$.
(2) f is continuous on $[0, \infty)$.
(3) f is discontinuous at $x = 1,2,3\ldots$
(4) f is continuous on [0,7].

51. Let be a real-valued monotone non-decreasing function on R
(1) for $a \in \mathbb{R}$ $\lim_{x \to a} f(x)$ exists.
(2) f is an unbounded function.
(3) $h(x) = e^{-f(x)}$ is a bounded function.
(4) if $a < b$, then $\lim_{x \to a} f(x) \le \lim_{x \to b} f(x)$

52. Let f be a continuous function on $\mathbb{R}$. Define $G(x) = \int_0^{\sin x} f(t)dt \ \forall x \in \mathbb{R}$.
Then
(1) $G'(x) = f(\cos x) \sin x$
(2) $G'(x) = -f(\sin x) \cos x$
(3) $G'(x) = f(\sin x) \cos x$
(4) $G'(x) = f(\sin x) \sin x$

53. Suppose f and g are differentiable on the interval $[a, \infty)$ such that. $f(a) \le g(a)$ and $f'(x) < g'(x) \forall x > a$. Then which of the following statements is true?
(1) $f(x) = g(x) \forall\ x \in [a, \infty)$
(2) $f(x) > g(x)$
(3) $f(x) < g(x)$
(4) None of the above

54. Which of the following statements are true?
(I) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto (0, 1)
(II) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\mathbb{R}$
(III) There exists a continuous function from $[0, \pi][2\pi, 3\pi]$ onto [0, 1]
(IV) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \right]$
(1) (I) and (II) (2) (II) and (III)
(3) (II) and (IV) (4) (I) and (IV)

55. The equation of the tangent plane to the surface: $z = 2x^2 - y^2$ are the point (1, 1, 1) is
(1) $x - y - 2z = 2$. (2) $4x - y - 3z = 1$.
(3) $2x - y - 2z = 1$ (4) $4x - 2y - z = 1$.

56. If $f: \mathbb{R} - \mathbb{R}$ *is* continuous function such that $f(x + y) = f(x) + f(y)$, for all $x, y = \mathbb{R}$, then

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(1) f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$.
(2) f is increasing if $f(1) \leq 0$ and decreasing if $f(1) \geq 0$
(3) f is a not an increasing function.
(4) f is neither an increasing nor a decreasing function

57. How many continuous real functions f can be defined on $\mathbb{R}$ such that $(f(x))^2 = x^2$ for every $x \in R$?
(1) Infinitely many (2) None
(3) 4 (4) 2

58. Let $f: \mathbb{R} - \mathbb{R}$ be a continuous function which takes irrational values at rational points and rational values at irrational points. Then which one of the following statements is true?
(1) f is uniformly continuous on $\mathbb{Q}$.
(2) f is uniformly continuous on $\mathbb{R}$
(3) f is uniformly continuous on $\mathbb{Q}^c$
(4) No such function exists.

59. The Maclaurin series expansion $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} -$ --is valid
(1) Only if $x \in [-1,1$ (2) $if\ x > -1$
(3) $only\ if\ x \in (-1,1]$. (4) for every $x \in \mathbb{R}$

60. Let f be a convex function with $f(0) = 0$. Then the function g defined on $(0, +\infty)$ as $g(x) = \frac{f(x)}{x}$
(1) is an increasing function
(2) is such that its monotonicity cannot be determined
(3) is neither increasing nor decreasing function.
(4) is a decreasing function

61. If α, β and γ are the roots of $x^3 + ax^2 + bx + c = 0$ then the value of $\alpha^2 + \beta^2 + \gamma^2$ is
(1) $a^2 - 2b$. (2) $b^2 - 2c$.
(3) $c^2 + 2a$ (4) $b^2 + 2c$

62. Consider the following statements
S_1: IF f is Riemann integrable in $[0,1]$ then f^2 is Riemann integrable in $[0,1]$.
S_1: $IF\ f^2$ is Riemann integrable in $[0,1]$ then f is Riemann integrable in $[0,1]$.
Then
(1) S_1 is true but S_2 is false
(2) S_1 is false but S_2 is true
(3) both S_1 and S_2 are false
(4) both S_1 and S_2 are true

63. The function $f(x) = \sin(x) + \cos(x)$ is
(1) increasing in $[0, \pi/2]$
(2) decreasing in $[0, \pi/2]$
(3) increasing in $[0, \pi/4]$ and decreasing in $[\pi/4, \pi/2]$
(4) Decreasing in $[0, \pi/4]$ and increasing in $[\pi/4, \pi/2]$

64. The graphs of the real valued functions $f(x) = 2\log(x)$ and $g(x) = \log(2x)$
(1) do not intersect
(2) intersect at one point only
(3) intersect at two points.
(4) intersect at more than two point

65. Let $f(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2$.
suppose $f(-1)0$ and $f(1) < 0$
(1) f has at least 3 real roots
(2) f has at most 3 real roots
(3) f has at most 1 real root
(4) all roots of f real

66. Let $x, y \in \mathbb{R}.$ if $|x-y| = |x| + |y|$ then
(1) $|x-y| = |x| - |y|$ (2) $|xy| = xy$
(3) $|x^2 + y| = |x^2| + |y|$ (4) $|x+y| = x + y$

67. Let $f: \mathbb{R} \to \mathbb{R}$ be a quadratic polynomial.Then which of the following is impossible ?
(1) $f(x) < f'(0), for\ all\ x \in \mathbb{R}.$
(2) $f'(x) > f(x)$, $for\ all\ x \geq 0.$
(3) $f'(0) = 0$ and $f(1) = f(4)$
(4) $f'(0) = 0$ and $f(x) \neq 0$ $for\ all\ x \in \mathbb{R}$

68. If α, β and γ are the roots of the polynomial $x^3 + x^2 + x + 1$, then the value of $\frac{1}{\alpha-1} + \frac{1}{\beta-1} + \frac{1}{\gamma-1}$ is
(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$
(3) $\frac{3}{2}$ (4) $-\frac{3}{2}$

69. Let $f\ \mathbb{R} \to \mathbb{R}$ be a function . which of these four statements mean that f is a constant function ?
(1) For all $x, y \in \mathbb{R}, f(x) = f(y)$.
(2) There exists $x \in \mathbb{R}$ $such\ that$ $for\ all\ y \in \mathbb{R}$ $f(x) = f(y)$.

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(3) There exists $x \in \mathbb{R}$ and there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$

(4) For each $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$.

70. Let P be a polynomial of degree 5 having 5 distinct real roots. Then

(1) the roots of P and P' occtu alternately.

(2) the roots of P' and P'' occur alternately.

(3) all the roots of P, P'', P'''', P'''' are real.

(4) it is possible to have a repeated root for P''''.

71. The value of the integral $\int_0^{10}(x - [x]dx$ is

(1) 2 (2) 3

(3) 4 (4) 5

72. Consider the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} -x+3 & x \in \mathbb{Q} \\ x^2 - 6x + 9 & x \in \mathbb{Q} \end{cases}$

The set of all points at which f is continuous is

(1) $\{2,3\}$ (2) $\{3\}$.

(3) $\mathbb{R} - \{2,3\}$ (4) $\mathbb{R} - \{3\}$.

73. Let $f: [0,1] \to \mathbb{R}$. $g: [0.1] \to \mathbb{R}$ given by $f(x) = \begin{cases} 1 & x \in \mathbb{Q} \cap [0,1] \\ 0 & x \notin \mathbb{Q} \cap [0,1] \end{cases}$ and $g(x) = \begin{cases} 1 & x \in \mathbb{Q} \cap [0,1] \\ 0 & x \notin \mathbb{Q} \cap [0,1] \end{cases}$ then

(1) both f and g are Riemann integrable.

(2) f is Riemann integrable but 9 is Not Riemann integrable.

(3) g is Riemann integrable but f is Not Riemann integrable

(4) both f and g are NOT Riemann integrable.

74. Consider $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin^3(|x|)$, then $f'(0)$

(1) is equal to-1 (2) is equal to 0

(3) is equal to 1 (4) does not exist

75. Consider the following two statements

S_1: If $f: [0.1] \to [0.1]$ is continuos then $\exists x_0 \in [0.1]$ such $f(x_0) = x_0$

S_2: There exists a continuous function $f: [0,1] \to [0,1] - \{\frac{1}{2}\}$ such that f is on to.

(1) Both, S_1 and S_2 are true.

(2) S_1 is true but S_2 is FALSE.

(3) S_2 is true but S_1 is FALSE.

(4) Both S_1 and S_2 are FALSE

76. If $|a - b| = |c - d|$, then

(1) $a = b + c - d$.

(2) $a = b - c + d$.

(3) $a = b + c - d$ and $a = b - c + d$.

(4) $a = b + c - d$ or $a = b - c + d$.

77. Consider $f: [-1,2] \to \mathbb{R}$ defined by $f(x) = \begin{cases} -x & If\ -1 \le x \le 0 \\ 2x^3 - 4x^2 + 2x, & If\ 0 < x \le 2 \end{cases}$ then the maximum value of $f(x)$ is equal to

(1) 0 (2) 2

(3) 4 (4) 10

78. The function eX from lR to lR is

(1) both one-one and onto.

(2) one-one but not onto.

(3) onto but not one-one.

(4) neither one-one nor onto.

79. The set of all real roots of the polynomial $P(x) = x^4 - x$ is

(1) $\{0,1\}$.

(2) the set of roots of $(x^2 - x)$.

(3) a set having four elements.

(4) an infinite set.

80. Let $f, g: \mathbb{R} \to \mathbb{R}$ be a polynomials with $\deg(f) = 5$ and $\deg(g) = 7$

Let $A = \{x \in \mathbb{R} : f(x) = g(x)\}$. Then

(1) $|A| \le 5$

(2) $|A| \le 7$

(3) A is finite, but $|A|$ can be arbitrarily large

(4) A can be finite

81. If one of the roots of the quadratic equation $10x^2 - 29x + (k + 4) = 0$ is the reciprocal, of the other root, then the value of k is

(1) 4 (2) 29

(3) 6 (4) – 3

82. What is the value of $\int_0^1 \int_y^1 e^{-x^2}\, dx\, dy$?

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(1) Undefined (2) $\sqrt{\pi}$

(3) $\frac{e+1}{2e}$ (4) $\frac{e-1}{2e}$

83. What is the volume of the region in the positive octant in $\mathbb{R}^3$ bounded by the planes $x = 0, y = 0, z = 0, x = 3$ and the parabolic cylinder $z = 4 - y^2$?

(1) 4 (2) 8

(3) 16 (4) ∞

84. Let r be a positive real number. Then, $\int_0^\infty \frac{dx}{1+e^{rx}}$ is equal to

(1) $\frac{\log 2}{r}$ (2) $\frac{1}{r}$

(3) $\frac{1}{er}$ (4) $\frac{1}{e^r}$

85. How many real roots does the polynomial $3X^5 + 9X + 7$ have?

(1) 1 (2) 3

(3) 5 (4) 0

86. What are the maximum and minimum of the function $f(x) = e^x - x$ on the interval $\left[-1, \frac{1}{2}\right]$?

(1) $1 + \frac{1}{e}$ and $\sqrt{e} - \frac{1}{2}$ (2) $1 + \frac{1}{e}$ and 1.

(3) $\sqrt{e} - \frac{1}{2}$ and $1 + \frac{1}{e}$. (4) $\sqrt{e} - \frac{1}{2}$ and $1 - \frac{1}{e}$.

87. What is the value of the integral $\int_0^{\frac{\log 3}{2}} \frac{dx}{e^{-x}+e^x}$?

(1) $\frac{\pi}{12}$

(2) $\log\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) - \log 2$

(3) $\sqrt{3} + \frac{1}{\sqrt{3}} - 2$

(4) $\frac{\pi}{6}$

88. Let X,Y and Z be finite sets and let $f: X \to Y$ and $g: Y \to Z$ be maps .

Which of the following assertions is always true?

(1) If gof is a bijection, then both g and f are bijections.

(2) If g is one to one, then gof is also one to one.

(3) If f is onto, then gof is also onto.

(4) If gof is onto, then $|Z| \leq |Y|$, where $|A|$ denotes the number of elements in any finite set A.

89. 100 women participate in a knockout tournament. On a knockout tournament a player is out of the tournament as soon as she loses one match. Some players might get byes in the first round.) How many matches will be held in the tournament?

(1) $\left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \cdots + \left[\frac{100}{2^6}\right]$

(2) 98

(3) 99

(4) 100

90. Consider the following assertions:

A. There exists an $x < 0$ such that $e^{\sin(2\pi)} = \frac{1}{2}$.

B. There exists an $x > 0$ such that $e^{\cos(2x)} = \frac{1}{6}$

C. There exists an $x \in \mathbb{R}$ such that $e^{|\sin(2x)|} = 2$.

Which of the above assertions is/are true?

(1) All of A, B and C are true. (2) A and C only

(3) B and C only (4) A and B only

91. Let $f: [-1,1] \to \mathbb{R}$ be a non constant function which is continuous at 0 with $f(0) = 1$ consider the following statements:

A. There exists $\epsilon > 0$ such that $f(x) > \frac{2020}{2021}$ for all $x \in$ $]0, \epsilon[$

B. There exists $\epsilon > 0$ such that $f(x) < \frac{2020}{2021}$ for all $x \in$ $]-\epsilon, 0[$.

Then the correct statement (s) is/are :

(1) A is true and B is false.

(2) B is true and A is false.

(3) Both A and B are true.

(4) Both A and B are false.

92. Let $f:]0, \infty[\to \mathbb{R}$ be defined as $f(x) = \frac{xe^{-x}}{1+x}$. Then

(1) f is a strictly increasing function on $]0, \infty[$

(2) f is a strictly decreasing function on $]0, \infty[$.

(3) there exists $x_0 \in]0, \infty[$ such that $f(x_0)f''(x_0) < 0$

(4) f is an onto function

93. For each $a = (a_0, a_1, a_2, a_3) \in \mathbb{R}^4$, let $f_a: \mathbb{R} \to \mathbb{R}$ be given by

$f_a(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$.

Consider the following statements

A. f_a is continuous at $x = 0$ for any $a \in \mathbb{R}^4$.

B. f_a is differentiable at $x = 0$ for any $a \in \mathbb{R}^4$.

Then

(1) A is true and B is false.

(2) both A and B are false.

(3) both A and B are true.

(4) B is true and A is false.

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94. Find the absolute maximum and absolute minimum of $f(x,y) = x^2 + y^2$ on the region defined by $x \geq 0, y \geq 0$ and $2x + y \leq 1$

(1) Absolute maximum is 2, absolute minimum is 0.
(2) Absolute maximum is 1, absolute minimum is 0.
(3) absolute maximum is 1/5, absolute minimum is 0.
(4) Absolute maximum is 1, absolute minimum does not exit.

95. Find the area of the region enclosed by the curve $y = x^3$ and the line $x = y$.

(1) ½ (2) 1/8
(3) 1/16 (4) 1

96. Find $\frac{df}{dx}$ where $f(x) = \int_0^{3x^2} t \sin(t) dt$.

(1) $3x^2 \sin 3x^2$ (2) $3x^2 \cos 3x^2$
(3) $12x^3 \sin 3x^2$ (4) $18x^3 \sin 3x^2$

97. Evaluate $\int_0^1 \int_0^1 2x^3 e^{x^2 y} dx dy$.

(1) $e^2 - e - 2$ (2) $e^2 - 2$
(3) $e - 2$ (4) 0

98. Let $f, g: \mathbb{R} \to \mathbb{R}$ be functions where $f(x) = e^{2x}$ and g is a polynomial of degree $n; n \geq 2$ Consider the following statement:

A. Cardinality of $\{x \in \mathbb{R}: f \circ g(x) = 1\} \leq n$.
B. if $g(0) = 1, g(1) = 5$, them there exists $x \in |0,1| such\ that\ f \circ g(x) = e^g$
C. The set $\{x \in \mathbb{R}: f \circ g(x = 1)\}$ is non-empty.

Which of the above statement is/are necessarily true;

(1) A and B only (2) A and C only
(3) Only A (4) B and C only

99. Let $p(x) = x^4 - 4x^3 + 2x^2 + ax + b$. Suppose that for every root λ of p, $\frac{1}{\lambda}$ is also a root of p. Find the values of a and b.

Answer: $a =$ ..... $b =$ .....

(1) 1 (2) 2
(3) 3 (4) 4

100. Pick out the true statements:

(1) Every polynomial of odd degree with real coefficients has a real root.
(2) Every non-constant polynomial with real coefficients can be factorized such that every factor has real coefficients and is of degree at most two.
(3) cannot be factorized
(4) None of the above

101. Let

$f(x) = \cos x - 1 + \frac{x^2}{2!}, x \in \mathbb{R}$.

Pick out the true statements:

(1) $f(x) \geq 0$ for $x \geq 0$ and $f(x) \leq 0$ for $x \leq 0$.
(2) f is a decreasing function on the entire real line
(3) F is increasing on entire real line
(4) None of the above

102. Let $p(x)$ be the polynomial $x^3 - 11x^2 + ax - 36$, where a is a real number. Assume that it has a positive root which is the product of the other two roots. Find the value of a.

(1) 24 (2) 36
(3) 46 (4) 56

103. Evaluate:

$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^3}}$.

(1) $e^{\frac{-1}{3}}$ (2) $e^{\frac{1}{6}}$
(3) e^{-1} (4) $e^{\frac{-1}{6}}$

104. Evaluate:

$\lim_{n \to \infty} \frac{1}{n^2} \sum_{k=1}^{n} \sqrt{n^2 - k^2}$.

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$
(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{8}$

105. Evaluate:

$\lim_{x \to 0} (1 - \sin x \cos x)^{cosec\ 2x}$.

(1) $e^{\frac{-1}{2}}$ (2) $e^{\frac{1}{2}}$
(3) e^{-1} (4) $e^{\frac{-1}{4}}$

106. Evaluate:

$\lim_{x \to \infty} \frac{1}{n^6} \sum_{k=1}^{n} k^5$.

(1) ¼ (2) 1/6
(3) 1/2 (4) 1/8

107. Evaluate:

$\lim_{x \to \infty} x \left(\log\left(1 + \frac{x}{2}\right) - \log\frac{x}{2}\right)$.

(1) e^2 (2) e
(3) e^{-2} (4) e^4

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108. Evaluate:

$\int_0^{\frac{\pi}{2}} \log \tan \theta \, d\theta$,

(1) 1 (2) 2

(3) 0 (4) 4

109. Let $f : \mathbb{R} \to \mathbb{R}$ be continuously differentiable. Evaluate:

$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f'\left(\frac{k}{n}\right)$.

(1) $f(1) - F(1)$ (2) $f(1) + F(0)$

(3) $f(1) + F(0)$ (4) $f(1) - F(0)$

110. Evaluate:

$\lim_{x\to 0} (1 + 3x^2)^{5 \cot x + 2\frac{\operatorname{cosec} x}{x}}$.

(1) e^2 (2) e

(3) e^{-2} (4) e^6

111. Pick out the true statements:

(1) $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

(2) $|\sin 2x - \sin 2y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

(3) $|\sin^2 x - \sin^2 y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

(4) None of the above

112. Let $x > 0$. Fill in the blanks with the correct sign $>, \geq, <$ or $\leq$:

(a) $\tan^{-1} x \ldots \frac{x}{1+x^2}$

(b) $\log(1 + x) \ldots \frac{x}{1+x}$

(1) $<$ (2) $\leq$

(3) $>$ (4) $\geq$

113. Which of the following statements are true?

(1) If $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{a_n}{n+1} = 0$, where $a_i \in \mathbb{R}$ for $0 \leq i \leq n$, then the polynomial $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ has a root in the interval $]0, 1[$.

(2) If $f : [a, b] \to \mathbb{R}$ is continuous and is differentiable in $]a, b[$, where $a > 0$ and if $\frac{f(a)}{a} = \frac{f(b)}{b}$. Then there exists $x_0 \in]a, b[$ such that $x_0 f'(x_0) = f(x_0)$.

(3) The polynomial $x^{13} + 7x^3 - 5$ has only one real root

(4) None of the above

114. Evaluate:

$\int_0^1 \max\left\{x, \frac{1}{2} - x\right\} dx$.

(1) $\frac{9}{16}$ (2) $\frac{3}{12}$

(3) $\frac{2}{8}$ (4) $\frac{4}{16}$

115. Evaluate:

$\lim_{\theta\to 0} (1 - 2\tan\theta)^{\cot\theta}$.

(1) e^2 (2) e

(3) e^{-2} (4) e^4

116. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Let $t \in \mathbb{R}$. Evaluate:

$\lim_{h\to 0} \frac{1}{h} \int_{t-4}^{t+h} f(s)\, ds$.

(1) $2f(t)$ (2) $2f$

(3) $2f^t$ (4) f

117. Solve the equation:

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

(1) $3 + 2\sqrt{2}, 2 + \sqrt{3}$ (2) $3 + 2\sqrt{2}, 2 \pm \sqrt{3}$

(3) $3 \pm \sqrt{2}, 2 \pm \sqrt{3}$ (4) $3 \pm 2\sqrt{2}, 2 \pm \sqrt{3}$

118. Evaluate:

$\lim_{x\to 0} (\cos x)^{\frac{1}{\sin^2 x}}$

(1) $e^{\frac{1}{2}}$ (2) e^2

(3) $e^{-\frac{1}{2}}$ (4) e^4

119. What are the maximum and minimum values in the region $\{(x, y) \mid x^2 + y^2 \leq 1, x + y \leq 0\}$ of the function $f(x, y) = x^2 + y$?

(1) $\frac{1}{2} + \frac{1}{\sqrt{2}}$ (2) $\frac{1}{2} - \frac{1}{2}$

(3) $\frac{1}{\sqrt{2}} + \frac{1}{2}$ (4) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

120. Let f be a continuous function from the real line $\mathbb{R}$ to the closed interval [1,3] such that:

- $f^{-1}(1)$ and $f^{-1}(3)$ are singletons, and
- $f^{-1}(x)$ consists of exactly two real numbers for every x in $(1,2) \cup (2,3)$.

Which of the following can be the cardinality of $f([2$

Choose all the correct options:

(1) 1 (2) 2

(3) countable infinity (4) uncountable infinity

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Answer Key									
1	2	3	4	5	6	7	8	9	10
3	2	2	1	1	2	4	2	2	3
11	12	13	14	15	16	17	18	19	20
3	1	1	1	4	3	1	2	2	3
21	22	23	24	25	26	27	28	29	30
4	1	2	2	3	4	4	3	2	1
31	32	33	34	35	36	37	38	39	40
2	1	2	1	1	3	3	1	3	4
41	42	43	44	45	46	47	48	49	50
1	3	4	4	2	3	3	2	3	3
51	52	53	54	55	56	57	58	59	60
4	3	3	3	4	1	3	4	3	1
61	62	63	64	65	66	67	68	69	70
1	1	3	2	2	4	4	4	4	1
71	72	73	74	75	76	77	78	79	80
4	1	3	4	4	1	3	2	3	2
81	82	83	84	85	86	87	88	89	90
3	4	3	1	1	2	1	4	3	1
91	92	93	94	95	96	97	98	99	100
1	2	1	2	1	4	3	1	1	3
101	102	103	104	105	106	107	108	109	110
1	2	4	1	1	2	1	3	4	4
111	112	113	114	115	116	117	118	119	120
1	3	1	1	3	1	4	3	4	4

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EXPLANATIONS

[CUET PG]

1. **Explanation:**

Q is not complex

Since I a sequence of rational number converges to an irrational number

$a_n = \left(1+\frac{1}{n}\right)^n$

$\lim_{n\to\infty} a_n = e$

$\because Q$ is not complete

Reason 'R' is False

2. **Explanation:**

$f: \mathbb{R} \to \mathbb{R}$

$f(1) = 3 \;\&\; f'(1) = 6$

$\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$ $(1^\infty - \text{ferm})$

$e^{\lim_{x\to 0}\left(\frac{1}{x}\left(\frac{f(1+x)}{f(1)}-1\right)\right)}$

$e^{\lim_{x\to 0}\frac{f(1+x)-f(1)}{xf(1)}}$

$e^{\lim_{x\to 0}\frac{f'(1+x)}{f(1)}}$

$e^{\lim_{x\to 0}\frac{f'(1)}{f(1)}}$

e^2

[CU-CET]

1. **Explanation:**

$L = \lim_{x\to 0}\frac{\log(\cos x)}{\sin^2 x} = \left(\frac{0}{0}\right)$ form

We apply L-Hospital rule

$= \lim_{x\to 0}\frac{\frac{1}{\cos x}\times -\sin x}{2\sin x\cos x}$

$= \lim_{x\to 0}\frac{-1}{2\cos^2 x} = -\frac{1}{2}$

$L = -1/2$

$\therefore$ limit exist it value $-1/2$

$\Rightarrow$ option (B) is right answer

2. **Explanation:**

$f(x) = \frac{1}{1+e^{1/x}} \qquad x \neq 0$

$\lim_{n\to 0^+}\frac{1}{1+e^{\frac{-1}{0+n}}} = \lim_{n\to 0}\frac{1}{1+\infty} = 0$

$\lim_{n\to 0^-}\frac{1}{1+e^{\frac{-1}{0+n}}} = \lim_{n\to 0}\frac{1}{1+e^{-1/n}} = \frac{1}{1} = 1$

$\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\Rightarrow$ option (4) is correct

3. **Explanation:**

$L = \lim_{x\to\infty} x\left(\log\left(1+\frac{x}{2}\right) - \log(x/2)\right)$

$= \lim_{x\to\infty}\frac{\log\left(\frac{1+x/2}{x/2}\right)}{1/x}$

$= \lim_{x\to\infty}\frac{\log\left(\frac{2}{x}+1\right)}{1/x}$ $\left(\frac{0}{0}\right)$ form

Apply L-Hospital's

$= \lim_{x\to\infty}\frac{\frac{1}{\frac{2}{x}+1}\times -2/x^2}{-1/x^2}$

$= \lim_{x\to\infty}\frac{2}{\frac{2}{x}+1}$

$L = 2$

$\Rightarrow$ option (1) is correct

4. **Explanation:**

Monotonic function can have at most countably infinite discontinuity

Monotonic function is continuous if it satisfies IVP.

$\therefore$ (a) $f(x) = [x]$ not always continuous

(b) cannot be nowhere continuous

(c) cannot be discontinuous at infinitely many point

$\Rightarrow$ is continuous only if it is satisfying IVP.

$\Rightarrow$ option (2) is correct

5. **Explanation:**

We know that every compact subset of set of real number is closed & bounded

$\Rightarrow$ option (3) is correct

6. **Explanation:**

The complement of set of real number is empty set which is both open as well as closed

So set of real number is open as well closed.

7. **Explanation:**

$\lim_{x\to 0^+}\frac{x-|x|}{x} = \lim_{x\to 0^+}\frac{x-x}{x} = 0$

$\lim_{x\to 0^-}\frac{x-|x|}{x} = \lim_{x\to 0^-}\frac{x-(-x)}{x} = 2$

Here $RHL = 0$ and $LHL = 2$

$\therefore RHL \neq LHL$

$\therefore$ limit does not exist

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[PRACTICE-QUESTIONS]

1. **Explanation:**

$f(x) = x^2(x-1) = x^3 - x^2$

$f(0) = 0$

$f(1) = 0$

$f'(x) = 3x^2 - 2x$

$f'(0) = 0$

(1) $f''(x) = 6x - 2 \neq 0$

(2) $f''(0) = 6(0) - 2 = -2 \neq 0$

(3) $f''(x) = 0$ for some $x \in (0,1)$

$6x - 2 = 0$

$x = \frac{1}{3} \in (0,1)$

$0 < \frac{1}{3} < 1$

$\therefore f''(x) = 0$ for some $x \in (0,1)$

$\Rightarrow$ option (3) is correct.

2. **Explanation:**

$f(x) = \begin{cases} 1 & for \quad x \in \mathbb{Q} \\ -1 & for \quad x \in \mathbb{Q}^c \end{cases}$

Here $[f(x)]^2$ is Riemann integrable on $[a,b]$ but $f(x)$ is not a Riemann integrable on $[a,b]$

$\therefore\Rightarrow$ option (2) is correct

3. **Explanation:**

$f(x) = \sqrt{x}\,\phi(x) = \frac{1}{\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{x}}\phi'(x) = \frac{-1}{2x\sqrt{x}}$ The given interval

$[1,2]\ a = 1\ b = 2$

Find the pt $c \in (1,2)$.

$\frac{f(b)-f(a)}{\phi(b)-\phi(a)} = \frac{f'(c)}{\phi'(c)}$

$\frac{\sqrt{2}-1}{\frac{1}{\sqrt{2}}-1} = \frac{\frac{1}{2\sqrt{c}}}{\frac{-1}{2c\sqrt{c}}}$

$\frac{\sqrt{2}-1}{1-\sqrt{2}} = \frac{1}{2\sqrt{2}} \times \frac{2c\sqrt{c}}{-1}$

$\frac{\sqrt{2}[\sqrt{2}-1]}{-[\sqrt{2}-1]} = -c$

$\Rightarrow C = \sqrt{2}$

$\Rightarrow$ option (2) is true.

4. **Explanation:**

$f(x) = x[x]$

$x \in (0,1) \rightarrow [x] = 0$

$x \in (1,2) \rightarrow [x] = 1$

$\int_0^2 x[x] = \int_0^1 x[x]dx + \int_1^2 x[x]dx$

$= \int_1^2 x\,dx = \left[\frac{x^2}{2}\right]_1^2 = \left[\frac{4}{2} - \frac{1}{2}\right] = \frac{3}{2}$

$\Rightarrow f$ is integrable on $[0,2]$ & $\int_0^2 f\,dx = \frac{3}{2}$

$\Rightarrow$ option (1) is correct

5. **Explanation:** for $x, y \in \mathbb{R}$ $\quad x \sim y \Leftrightarrow 3 + 4x = 2y$

(1) $1 \sim \frac{7}{2}$ $\quad 3 + 4(1) = 2\left(\frac{7}{2}\right)$

$7 = 7$

(2) $2 \sim \frac{1}{7}$ $\quad 3 + 4(2) = 2\left(\frac{1}{7}\right)$

$11 \neq \frac{2}{7}$

(3) $-1 \sim \frac{7}{2}$ $\quad 3 + 4(-1) = 2\left(\frac{7}{2}\right)$

$-1 \neq 7$

(4) $-2 \sim \frac{1}{7}$ $\quad 3 + 4(-2) = 2\left(\frac{1}{7}\right)$

$-5 \neq \frac{2}{7}$

Option (1) is correct

6. **Explanation:** $F : [-1,1] \rightarrow \mathbb{R}$ defined

$F(x) = \begin{cases} 2 & x \neq 0 \\ 0 & x = 0 \end{cases}$

Which is a Riemann integral exist but not continuous on [a, b]

$\Rightarrow$ Every function for which Riemann integrable exists is continuous on [a, b] is False

$\Rightarrow$ option (2) is correct

7. **Explanation:** A function $F : [-1,1] \rightarrow \mathbb{R}$ is defined as

$F(x) = \begin{cases} 2 & x \neq 0 \\ 0 & x = 0 \end{cases}$

Given function discontinuous at $x = 0$ which is finite discontinuity

$\Rightarrow$ F is Riemann integrable over [-1, 1]

8. **Explanation:** if $F(4) = 4$ $\qquad F'(4) = 1$

$\lim_{x \to 4} \frac{2 - \sqrt{F(x)}}{2 - \sqrt{x}} = \lim_{x \to 4} \frac{2 - \sqrt{4}}{2 - 2} = \frac{0}{0}$ from

Apply L – Hospital Rule

$= \lim_{x \to 4} \frac{0 - \frac{1}{2\sqrt{f(x)}} F'(x)}{\frac{1}{2\sqrt{x}}}$

$= \frac{-\frac{1}{2\sqrt{4}}(1)}{\frac{1}{2\sqrt{4}}}$

$\frac{1}{2\sqrt{4}} \times \frac{2\sqrt{4}}{1} = 1$

$\Rightarrow$ option (2) is correct

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9. Explanation:

$\lim_{x\to 4} \frac{2-\sqrt{f(x)}}{2-\sqrt{x}} = \frac{2-\sqrt{f(4)}}{2-\sqrt{4}}$

$\frac{0}{0}$ form

by l hospital rule

$\lim_{x\to 4} \frac{f'(x)}{2\sqrt{f(x)}} \times 2\sqrt{x} = 1$

10. Explanation: Maximum value of $\left(\frac{1}{x}\right)^x \quad x > 0$

$y = \left(\frac{1}{x}\right)^x$

Taking log on both side

$\log y = \log\left(\frac{1}{x}\right)^x$

$\log y = x\log\left(\frac{1}{x}\right)$

$\log y = x(\log 1 - \log x)$

$\log y = x(0 - \log x)$

$\log y = -x\log x$

Diff w.r.t x

$\frac{1}{y}\frac{dy}{dx} = -\frac{x}{x} - \log x$

$\frac{1}{y}\frac{dy}{dx} = -1 - \log x$

$\frac{dy}{dx} = y(-1 - \log x)$

$\Rightarrow \frac{dy}{dx} = 0$

$y(-1 - \log x) = 0$

$-1 - \log x$

$\log x = -1$

$x = e^{-1}$

$y = \left(\frac{1}{x}\right)^x \Rightarrow \left(\frac{1}{e^1}\right)^{e^1} = e^{1/e}$

$\Rightarrow$ option (3) is correct

11. Explanation:

$\frac{\partial^2 f}{\partial x^2} = 6xy$

$\frac{\partial^2 f}{\partial y^2} = 0$

$\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ are equal at X-axis and at Y-axis

12. Explanation:

$$x\frac{\partial}{\partial x}f(x,y) + y\frac{\partial}{\partial x}f(x,y) = p, f(x,y)$$

13. Explanation:

f and g are one-one onto $\Rightarrow g$ of is one-one onto

14. Explanation:

x is a real number $s.t\ |1-x| = x-1$

$|1-x| = \begin{cases} 1-x & x \geq 1 \\ -(1-x) & x < 1 \end{cases}$

$= \begin{cases} 1-x & x \geq 1 \\ x-1 & x < 1 \end{cases}$

$\Rightarrow |1-x|$

$x \geq 1 \quad 1-x$

$x < 1 \quad -(1-x)$

Case 1 : $x \geq 1$

$|x-x| = x-1$

$1-x = x-1$

$1+1 = x+x$

$x = 1$

Case 2 : $x < 1$

$|1-x| = x-1$

$-(1-x) = x-1$

$x-1 = x-1$

$\therefore x$ less than 1 satisfies the eq.

$\Rightarrow x < 1$

$\Rightarrow$ option (1) is correct

15. Explanation: $f(x) = x^{\frac{3}{2}}$

$\sqrt{x^3} \geq 0, x^3 \geq 0$

$\therefore$ Domain : $[0,\infty)$

Range : $[0,\infty)$

$\therefore$ option (4) is correct

16. Explanation:

$f(x) = 2|x| + 1$

$f(-x) = 2|x| + 1$

$f(-x) = 2|x| + 1$

$f(-x) = f(x)$ $f(x)$ is even function

$g(x) = \frac{1}{x^2-1}$

$g(x) = \frac{1}{(-x)^2-1}$

$= \frac{1}{x^2-1}$

$g(x) = g(x)$ $g(x)$ is even function

$\therefore f\phi g$ both even functions.

$\Rightarrow$ option (3) is correct.

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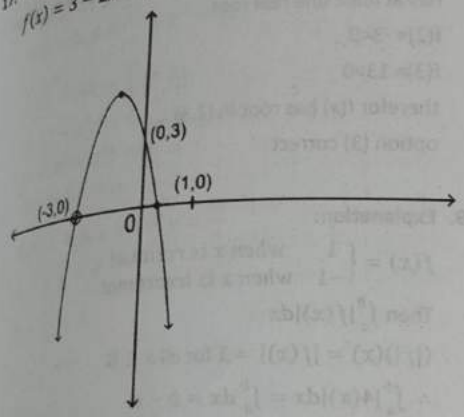
17. **Explanation:**

$f(x) = 3 - 2x - x^2$



⇒ Option (1) is correct

18. **Explanation:**

$f(x) = x^3 - x - 1$

$f(1) = -1\ f(2) = 2^3 - 2 - 1 = 5$

∴ [1,2] has intersect $bet^n - 1$ to 5.

∴ f has zero between the interval [1,2]

⇒ option (2) is correct

19. **Explanation:**

Let $L : \mathbb{R} \to \mathbb{R}$ be linear function $s.t. f(1) = \alpha\ f(x) =$ cx for some $c \in \mathbb{R}$.

$f(1) = \alpha$

$\alpha = c(1)$

$\alpha = c$

$f(x) = \alpha x$

$f'(x) = \alpha \forall x \in \mathbb{R}$

∴ f is continuous & differentiable and $f'(x) = \alpha$ for all $x \in \mathbb{R}$

⇒ option (2) is correct

20. **Explanation:**

Every bounded function is noted not be Rieman integrable.

Example $f(x) = \begin{cases} 1 & if\ x\ rational \\ 0 & otherwise \end{cases}$

⇒ option (3) is correct

21. **Explanation:**

F(x) is not differentiable at 0,1

22. **Explanation:**

$f : [a,b] \to R\ g : [a,b] \to R$ are two function such that $f'(x) = g'(x)\ \forall\ x \in [a,b]$

f' & g' denote first derivative of f & g respectively.

$(f-g)(x)\ \forall x \in [a,b]$

$f'(x) - g'(x) = 0$

$\Rightarrow (f' - g')(x) = 0$

$\frac{d}{dx}(f-g)(x) = 0$

f must be constant

⇒ option (1) is correct

23. **Explanation:**

(2) $\sqrt{AB}$

24. **Explanation:**

$f : [0,1] \to \mathbb{R}$ is defined $f(x) = 2\alpha x$

$\int_0^1 f(x)dx = \sum_{r=1}^{\infty} \int_{\frac{1}{r+1}}^{\frac{1}{r}} f(x)dx$

$= \sum_{r=1}^{\infty} \int_{\frac{1}{r+1}}^{\frac{1}{r}} 2rxdx$

$= \sum_{r=1}^{\infty} \frac{2r+1}{r(r+1)^2}$

$= \sum_{r=1}^{\infty} \left(\frac{1}{(r+1)^2} + \frac{1}{r} - \frac{1}{r+1}\right)$

$\Sigma\left(\frac{1}{r} - \frac{1}{r+1}\right) + \Sigma\frac{1}{(r+1)^2}$

$= \left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n}\right)\right]$

$\left[\frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}\right]$

$= \left(1 - \frac{1}{n}\right)\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}\right] - 1$

Taking $n \to \infty$ we get.

$\int_0^1 f(x)dx = 1 + \left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \infty\right] - 1$

$= 1 + \frac{\pi^2}{6} - 1$

$= \frac{\pi^2}{6}$

Option (2) is correct

25. **Explanation:**

$\int_0^1 x^m(1-x)^n dx$ exist if.

$= -\frac{1}{m+1}\int_0^1 (1-x)^n dx^m$

$= \frac{1}{m+1}\int_0^1 x^{m+1} d(1-x)^n$

$= \frac{n}{m+1}\int_0^1 (1-x)^{m-1} x^{m+1} dx$

This integration exist if m & n are positive integer.

$m > -1, n > -1$

⇒ option (3) is corr

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26. Explanation:

$y = \left[\log\left(x + \sqrt{(1+x^2)}\right)\right]^2$

$y' = \frac{2\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$

$\sqrt{1+x^2}\, y' = 2\log(x + \sqrt{1+x^2})$

$(1+x^2)y^2 = 4y$

$\Rightarrow (1+x^2)\,(2y'y'') + (2x)\,(y')^2 = 4y'$

$\Rightarrow (1+x^2)\,(2y'') + 2xy' = 4$

$\Rightarrow 2(1+x^2)y'' + 2xy' - 4 = 0$

Leibnitz rule

$(y.f(x))_n = nc_o\, y_n f(x) + nc_1 y_{n+1} f'(x) + \cdots +$

$nc_n\, y_o f^n(x)$

So, $\{(1+x^2)y_2\}_n = nc_o\, y_{n+2}\,(1+x^2) +$

$nc_1\, y_{n+1}(2x) + nc_2\, y_n(2) + nc_3\, y_{n-1}0 + \cdots + 0 + 0$

$\therefore = y_{n+2}(1+x^2) + n\, y_{n+1}(2x) + n\frac{(n-1)}{2} y_n(2).$

Now

$(2xy_1)_n = nc_o\, y_{n+1}(2x) + nc_1\, y_n(2) +$

$nc_2\, y_{n-1}(0) + 0 + \cdots + 0$

$(2xy_1)_n = y_{n+1}(2x) + ny_n(2).$

$2\, y_{n+2}(1+x^2) + 4x.x\, y_{n+1} + 2n(n-1)\, y_n +$

$y_{n+1}(2x) + 2ny_n = 0.$

At $x = 0$

$\Rightarrow 2\, y_{n+2} + 2n\,(n-1)\, y_n + 2ny_n = 0$

$\Rightarrow y_{n+1} + n^2 y_n = 0$

$\Rightarrow y_{n+2}(0) = -n^2\, y_{n(0)}$

$\Rightarrow$ option (4) is correct

27. Explanation:

For $x > 0\ 0 < \theta < 1$

$\log_{10}(x+1) = f(x)$

$\frac{f(x)-f(0)}{x-0} = f'(\theta x)$

$\Rightarrow \frac{\log_{10}(1+x)}{x} = \frac{1}{1+\theta x}$

Now $0 < \theta < 1\ x > 0 \Rightarrow \theta x < x$

$1 + \theta x < 1 + x$

$\frac{1}{1+\theta x} > \frac{1}{1+x}$

$\frac{x}{1+\theta x} > \frac{x}{1+x}$ _____ (ii)

Again $0 < \theta < 1\ x > 0$

$1 + \theta x > 1$

$\frac{1}{1+\theta x} < 1$

$\frac{x}{1+\theta x} < x$ _____ (ii)

From (1) & (ii) we get.

$\Rightarrow \frac{x}{1+x} < \log_{10}(1+x) < x$

$\Rightarrow$ option (4) is correct.

28. Explanation:

As $f(x) = x^3 - 3x - 5$ is odd degree polynomial so it has at least one real root

f(2)= -3<0

f(3)= 13>0

therefor f(x) has root in (2,3)

option (3) correct

29. Explanation:

$f(x) = \begin{cases} 1 & when\ x\ is\ rational \\ -1 & when\ x\ is\ irrational \end{cases}$

Then $\int_a^b |f(x)|dx$

$(|f|)(x) = |f(x)| = 1$ for all $x \in \mathbb{R}$

$\therefore \int_a^b |4(x)|dx = \int_a^b dx = b - a$

$\Rightarrow$ option (2) is correct

30. Explanation:

A function f is defined $[0,1]$

$f(x) = \begin{cases} \frac{p}{q} & xe(Q - [0]) \cap [0,1] = A \\ 0 & xe\ (Q^c \cup [0]) \cap [0,1] = B \end{cases}$

$[0,1] = A \cup B$

$\int_{[0,1]} f(x)dx = \int_A f(x)dx + \int_B f(x)dx$

$= f(x)m(A) + f(x)m(B)$ -------- (1)

$= \frac{p}{a} m(A) + 0.m(B)$

$m(Q^* \cap [0,1]) = m(A) = 0.$

Q^* is countable its measure is 0

(1) becomes

$= 0$

$\Rightarrow$ option (1) is correct

31. Explanation:

$f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

$f_x = 2x - \frac{2}{x^2=0}\, f_y = 2y - \frac{2}{y^2} = 0$

$\Rightarrow$ find the $pt(x,y)$.

$\Rightarrow 2x^3 - 2 = 0 \Rightarrow 2y^3 - 2 = 0$

$2x^3 = 2 \Rightarrow 2y^3 = 2$

$x^3 = 1\ y^3 = 1$

$f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

$f(1,1) = 1 + 1 + 2 + 2 = 6 > 0$ is minimum at (1,1)

$\Rightarrow$ option (2) is correct

32. Explanation:

$x^3 + y^3 - 3axy.$

$\frac{\partial f}{\partial x} = 3x^2 - 3ay = 0\ \frac{\partial f}{\partial y} = 3y^2 - 3ax = 0$

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$\Rightarrow 3x^2 = 3ay \;\; 3y^2 - 3ax = 0$

$\Rightarrow x^2 = ay \Rightarrow y^2 = ax$

Now sub. $y = \frac{x^2}{a}$ in $y^2 = ax$.

$\left(\frac{x^2}{a}\right)^2 = ax \Rightarrow \frac{x^4}{a^2} = ax$

$\Rightarrow x^4 - a^3x = 0$

$\Rightarrow x(x^3 - a^3) = 0$

$\Rightarrow x = 0 \; x = a$

Subs $x = a, x = 0$ in $y = \frac{x^2}{a}$

$x = a \;\; x = 0$

$y = \frac{a^2}{a} = a$

$y = \frac{0}{a} = 0$

$pt(a, a)$

$pt(0,0)$

Now diff. again with respective to x & y

$\frac{\partial^2 f}{\partial x^2} = 6x \Rightarrow \left(\frac{\partial^2 f}{\partial x^2}\right)_{(0,0)} = 0$

$\Rightarrow \left(\frac{\partial^2 f}{\partial x^2}\right)_{(a,a)} = 6a$

$\frac{\partial^2 f}{\partial y^2} = 6y \Rightarrow \left(\frac{\partial^2 y}{\partial y^2}\right)_{(0,0)} = 0 \Rightarrow \left(\frac{\partial^2 f}{\partial y^2}\right)_{(a,a)} = 6a$

$\frac{\partial^2 f}{\partial x \partial y} = -3a \Rightarrow \left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(0,0)} = -3a$

$\Rightarrow \left(\frac{\partial^2 y}{\partial x \partial y}\right)_{(a,s)} = -3a$

$D = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$

$D_{(0,0)} = 0 - (-3a)^2 = -9a^2 < 0$

$D(a, a) = 6a(6a) - (-3a)^2 = 27a^2 > 0$

Now $D_{(0,0)} < 0 \rightarrow$ saddle ptt

$D_{(a,a)} > 0 \rightarrow pt$ for minima.

$\Rightarrow$ option (1) is correct

33. Explanation:

By mean value then

$F'(c) = \frac{F(B) - F(a)}{b - a}$ _____ (1)

$f(x) = x(x - 2) \; a = 0 \; b = 1$

$x^2 - 2x$

Equation (1) becomes

$2c - 2 = \frac{(1-2)-0}{1-0}$

$2c - 2 = -1$

$2c = -1 + z$

$2c = 1$

$\Rightarrow C = \frac{1}{2}$

$\Rightarrow$ option (2) is correct

34. Explanation: The Taylor series of $cos(x)$ around $a = -\frac{\pi}{4}$, up to $n = 3$ is

$cos(x) \approx P(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x + \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x + \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x + \frac{\pi}{4}\right)^3$

35. Explanation:

Since $f: A \rightarrow A$ and $B \subset A$

Let, $x \in B$ is arbitrary. Then $f(x) \in f(B)$

[$f(B)$ denote the set of all images of elements of B.]

$\Rightarrow x = f^{-1}(f(x)) \in f^{-1}(f(B))$

(as f^{-1} is also defined from A to A)

Since $x \in B$ was arbitrary:

we have $B \subset f^{-1}(f(B))$

Hence option (1) is correct.

36. Explanation:

$f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \sin(x^3) \; \forall x \in \mathbb{R}$

clearly $f(x)$ is continuous function as it is composition of two continuous function $g(x) = \sin x$ and $h(x) = x^3$ on $\mathbb{R}$.

also $f(x)$ is differentiable on $\mathbb{R}$ as $f(x) = 3x^2 cos(x^3) \forall x \in \mathbb{R}$ is well-defined.

Now we have to check the uniform continuity of $\sin(x^3)$,

Let us assume that $X_n = \left(n\pi + \frac{\pi}{2}\right)^{\frac{1}{3}}$

Then $X_{n+1} - X_n = \left((n+1)\pi + \frac{\pi}{2}\right)^{\frac{1}{3}} - \left(n\pi + \frac{\pi}{2}\right)^{\frac{1}{3}} \rightarrow 0 \; as \; n \rightarrow \infty$

But $f(X_{n+1}) - f(X_n) = \sin\left((n+1)\pi + \frac{\pi}{2}\right) - \sin\left(nr - \frac{\pi}{2}\right)$

Hence $f(X_{n+1}) - f(X_n)$ does not converges to zero.

Therefore $\langle X_n \rangle$ is a cauchy sequence $\langle f(x_n) \rangle$ but is not cauchy.

$= f(x) = sin(x^3)$ is not uniformly continuous.

Hence option (3) is correct

37. Explanation: If f is a real valued function, then we define,

$$f(x) = \begin{cases} -1 & 0 \le x \le \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \le x \le 1 \end{cases}$$

Then it does not satisfy any of the options.

In the statement of question f must be continuous function on [0, 1] and differentiable on (0, 1), then by [illegible] theorem; $\exists \; t \in (0,1)$ such that

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$f'(t) = \frac{f(-1)-f(0)}{1-0} = \frac{\frac{1}{2}-(-1)}{1} = \frac{3}{2}$

Hence option (3) is correct.

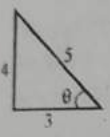
38. Explanation: Since $3^x + 4^x = 5^x$

$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x = 1$

Let us consider $\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$, then

$\cos^x\theta + \sin^x\theta = 1$

Consider a right angle triangle as then $\cos\theta$ and $\sin\theta$ are the ratio of base and perpendicular with hypotenuse of right angle triangle.



Hence $\cos^2\theta + \cos^2\theta = 1$

$\therefore x = 2$ is only positive solution.

Hence option (1) is correct.

39. Explanation: For options (A) and (B), let us take an example, as $f(x) = x \forall x \in \mathbb{R}$, then $f'(x) = 1 \forall x \in \mathbb{R}$

Clearly $f(x)$ is bounded, but $f(x)$ do not have maximum in $\mathbb{R}$.

Given that $f: \mathbb{R} \to \mathbb{R}$ is differentiable, hence continuous and $f'(x)$ is bounded on $\mathbb{R}$ implies that $f(x)$ satisfies the Lipschitz continuity on $\mathbb{R}$, hence f is uniformly continuous on $\mathbb{R}$

Hence option (3) is correct.

40. Explanation: Given that,

$\int_0^{\sin x} f(t)dt = \frac{\sqrt{3}x}{2}, 0 \le x \le \frac{\pi}{2}$

Using, Leibnitz rule, differentiating both side we have,

$f(\sin x)\cos.x = \frac{\sqrt{3}x}{2}$

At $x = \frac{\pi}{6}$;

we have $f\left(\sin\left(\frac{\pi}{6}\right)\right).\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}x}{2}$

$\Rightarrow f\left(\frac{1}{2}\right).\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

i.e. $f\left(\frac{1}{2}\right) = 1$

Hence option (4) is correct.

41. Explanation:

$f: [0,1] \to [0,1]$ is defined as

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Continuity at (0):

$\therefore \lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^2 \sin\left(\frac{1}{x}\right)$

Since, $\left|\sin\left(\frac{1}{x}\right)\right| \le 1 \forall x$,

we have, $-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2 \forall x$

Therefore by sand witch theorem,

we have $\lim_{x\to\infty} x^2 \sin\left(\frac{1}{x}\right) = 0$

Hence $f(x)$ is continuous over [0, 1].

Since every continuous function over closed and bounded interval is uniformly continuous, so $f(x)$ is uniformly continuous.

$\therefore$ Also, $f'(x) = 2x \sin\left(\frac{1}{x}\right) + x^2.\cos\left(\frac{1}{x}\right).\left(\frac{-1}{x^2}\right)$

$= 2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ is defined for all $x \in (0,1)$

Now at $x = 0$ we have $f(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} =$

$\lim_{h\to 0} h \sin\left(\frac{1}{h}\right)$

Again using sand witch theorem,

we have $\lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$

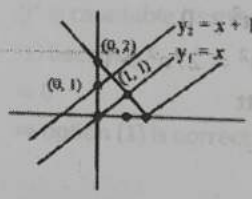
Hence $f(x)$ is differentiable at 0.

Hence option (1),(2) and (4) are correct.

42. Explanation:

$f: \mathbb{R} \to \mathbb{R}$ is defined as

$f(x) = \min\{x, x+1, |x-2|\}$



$$\therefore f(x) = \begin{cases} x & : x \le 1 \\ 2-x & : 1 \le x \le 2 \\ x-2 & : x \ge 2 \end{cases}$$

Clearly, f increases in the interval $(-\infty, 1]$ and decreases on the interval $[1, 2]$

continuity at $x = 1, x = 2$:

At $x = 1, f(1) = 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2-x) = 1$

continuous at $x = 1$

At $x = 2, f(2) = 2 - 2 = 0$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2-x) = 0$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x-2) = 1$

$\therefore$ continuous at $x = 2$.

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Now we check the differentiability at $x = 1$ and at $x = 2$, we have,

$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{2-(1+h)-1}{h} = -1$

$Lf'(1) = \lim_{h\to 0} \frac{f(1)-f(1-h)}{h} = \lim_{h\to 0} \frac{1-(1+h)}{h} = 1$

Clearly, $Rf'(1) \neq Lf(1)$

At $x = 2$;

$Rf'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{(2+h)-2-0}{h} = 1$

$Lf'(2) = \lim_{h\to 0} \frac{f(2)-f(2-h)}{h} = \lim_{h\to 0} \frac{0-2+(2-h)}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$

Clearly, $Rf(2) \neq Lf(2)$

Hence not differentiable at $x = 1$ and $x = 2$

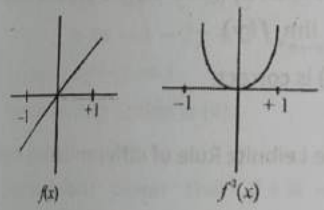
Hence options (1), (2), (3) and (4) are correct.

43. **Explanation:** Given that $f: 1 \to \mathbb{R}$ be an increasing function.

Let us consider $I(-1,1)$ and $f(x) = x \forall x \in I$, then clearly $f(x)$ is an increasing function on I

But $f^2: I \to \mathbb{R}$ is defined as $f^2(x) = (f(x))^2 = x^2 \forall x \in I = \{-1,1\}$

Which is neither increasing nor decreasing on I.



Hence, option (4) is correct.

44. **Explanation:** Suppose that $x \in [0,1]$ be arbitrary. let us define a sequence $\langle x_n \rangle$ by setting.

$x_1 = x, x_2 = x_1^2, x_3 = x_2^2, \ldots . x_{n+1} = x_n^2 \ldots .$

Then this sequence $\langle x_n \rangle$ converges to 0, and for $\cap n \in \mathbb{N}$ we have

$f(x_{n+1}) = f(x_n^2) = f(x_n)$ for all $n \in \mathbb{N}$

Then by induction, we have $f(x_n) = f(x) \forall n \in \mathbb{N}$

Also $\langle x_n \rangle$ converges to 0 and f is continuous function Then

$\lim_{n\to\infty} f(x_n) = f\left(\lim_{n\to\infty} x_n\right) = f(0)$

i.e., $f(x) = f(0)$

Since $x \in [0,1)$ was arbitrary.

Hence $f(x) = f(0)$ for all $x \in [0,1)$

Also, F is continuous at $x = 1$, so we must have $f(1) = f(0)$.

Hence f is constant. Therefore being constant it is differentiable and uniformly continuous

Hence option (4) is correct.

45. **Explanation:** Note that any continuous function $f: [a,b] \to [a,b]$ must have at least one fixed point. $i.e.\ f(x) = x$ for at least one $x \in [a,b]$.

Let $I: [a,b] \to [a,b]$ be identity function then $I(x) = x\ \forall x \in [a,b]$

$\therefore f$ and I agree at least at one point

Hence option (2) is correct.

46. **Explanation:**

$h(x) = x[x] = \begin{cases} (n-1)x & : n-1 \leq x < n \\ nx & : n \leq x < n+1 \end{cases}$

where $n \in \mathbb{Z}$,

Then $\lim_{x\to n^-} f(x) = \lim_{x\to n^-} (n-1)x = n^2 - n$ and

$\lim_{x\to n^+} f(x) = \lim_{x\to n^+} nx = n^2$

$\therefore h(x)$ is continuous at n if

$\lim_{x\to n^-} f(x) = \lim_{x\to n^+} f(x) = n^2$

$i.e.\ n^2 - n = n^2$

$i.e.\ n = 0$

$\therefore h(x)$ is continuous if $x \neq \pm 1, \pm 2, \ldots .$

Hence option (3) is correct

47. **Explanation:**

$g: \mathbb{R} \to \mathbb{R}$ is defined as

$g(x) = \begin{cases} 0 & if\ x \geq 0 \\ e^{-\frac{1}{x^2}} & if\ x < 0 \end{cases}$

Clearly, $g(0) = 0 = \lim_{x\to 0^+} g(x)$

Also, $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} e^{\frac{-1}{x^2}} = e^{-\infty} = 0$

Hence g is continuous at $x = 0$

Also, the right hand derivative at of all order 0 is 0 as function is constant there. Also, the left hand derivative at 0 is given

$g'(x) = e^{-\frac{1}{x^2}} \cdot \frac{2}{x^3}$

$g''(x) = \frac{4}{x^6} . e^{\frac{-1}{x^2}} - \frac{6}{x^4} . e^{\frac{-1}{x^2}} = \left(\frac{4}{x^6} - \frac{6}{x^4}\right) . e^{\frac{-1}{x^2}}$

Hence by induction, we have

$g^n(x) = \mathbb{Q}_n\left(\frac{1}{x}\right) . e^{\frac{-1}{x^2}}$, where $\mathbb{Q}_n$ be a Polynomial

$\therefore g^{(n)}(0) = \lim_{x\to 0} \mathbb{Q}_n\left(\frac{1}{x}\right) . e^{\frac{-1}{x^2}}$

$= \lim_{h\to\infty} \mathbb{Q}_n(h) . e^{-h^2}$ where $h = \frac{1}{x}$

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Hence $g^n(0) = 0$ for all n.

Hence the taylor series of $g(x)$ about $x = 0$, is identically zero (as all derivatives are zero). Which is not equals to g.

Hence option (3) is correct.

48. Explanation: Note that if f and g are uniformly continuous function on $I \subseteq \mathbb{R}$, Then $f + g$ is also uniformly on $\mathbb{R}$. Let $f(x) = x \sin x$ and $g(x) = \frac{1}{1+x^2}$

Note that $g(x) = \frac{1}{1+x^2}$ is continuous for all $x \in \mathbb{R}$, and both $\lim_{x\to\infty} f(x) = 0 = \lim_{x\to-\infty} f(x)$, exist, and finite.

Hence g is uniformly continuous over $\mathbb{R}$.

Also $f(x) = x \sin x$ is continuous for all $x \in \mathbb{R}$. But it is uniformly continuous only over a closed and bounded interval.

Hence $f(x) + g(x)$ is uniformly continuous, if I is compact.

Hence option (2) is correct

49. Explanation: $f : \mathbb{R} \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} x^2 \text{ if } x \in (0,2) \cap \mathbb{Q} \\ 2x - 1 \text{ if } x \in (0,2) \cap (\mathbb{R}/\mathbb{Q}) \end{cases}$$

Note that this function is continuous only

When $x^2 = 2x - 1$

i.e. $x^2 - 2x + 1 = 0$

i.e. $(x-1)^2 = 0$ *i.e.* $x = 1$

$\Rightarrow$ function is continuous at $x = 1$.

Now we check the differentiability only at $x = 1$, then if x approaches to 1 through rationals we have,

$$f'(1) = \lim_{x\to1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to1} \frac{x^2-1}{x-1} = 2$$

Also, if x approaches to I through irrationals, we have

$$f(1) = \lim_{x\to1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to1} \frac{x^2-1}{x-1} = 2$$

Hence function is differentiable at $x = 1$.

Hence option (3) is correct

50. Explanation:

$f : [0,\infty) \to \mathbb{R}$ is defined as $f(x) = [x]$

Therefore each $n \in \mathbb{N}$, we have,

$$f(x) = \begin{cases} n-1 & n-1 \le x < n \\ n & n \le x < n+1 \end{cases}$$

$\therefore \lim_{x\to n^-} f(x) = n - 1$ and $\lim_{x\to n^+} f(x) = n$

And clearly, $n - 1 \neq n$,

Hence it is discontinuous for all $x \in \mathbb{N}$.

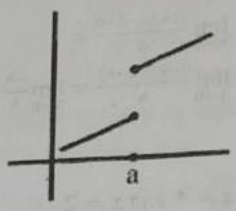
Hence option (3) is correct

51. Explanation: Given that f be a real valued monotone non-decreasing function on $\mathbb{R}$.

i.e. if $x_1 \le x_2$ then $f(x_1) \le f(x_2)$

Option (A) is not true as function in the graph have no limit at 'a'.

We may also define, $f(x) = \begin{cases} 1 & : x \in (0,\infty) \\ 0 & : x \in (\infty, 0) \end{cases}$



Option (B) not necessarily, as a constant function is non-decreasing bounded function.

Option (C) given that, $h(x) = e^{-f(x)}$

Let $f(x) = x$, clearly $f(x)$ is non – decreasing function,

Then, $\lim_{x\to-\infty} h(x) = \lim_{x\to-\infty} e^{-x}$ do not exist.

Clearly option (4) is correct, as for $x \le a < b \le y$, we have, $f(x) \le f(b) \le f(y)$

$\therefore \lim_{x\to a^-} f(x) \le f(a)$ and $f(b) \le \lim_{y\to b^+} f(y)$

(as every x is less than a) (as every y is greater than a)

$\Rightarrow \lim_{x\to a^-} f(x) \le \lim_{x\to b^+} f(y)$

Hence option (4) is correct.

52. Explanation: The Leibnitz Rule of differentiation is given as

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(x,t)dt = f\big(x,v(x)\big)v'(x) - f\big(x,u(x)\big)u'(x) + \int_{u(x)}^{v(x)} \frac{\partial f}{\partial x} dt$$

Using this, we have,

$$G'(x) = f(\sin x).\frac{d}{dx}\sin x - f(0).\frac{d}{dx}.0$$

$$= f(\sin x)\cos x$$

Hence option (3) is correct.

53. Explanation: Let us assume that, $h(x) = g(x) - f(x)\ \forall x \in [a,\infty)$

Then $h(a) = g(a) - f(a) \ge 0$

Also, $h'(x) = g'(x) - f'(x) > 0, \forall x > a$

Since $h'(x) > 0$, hence it is monotonically increasing function, and $h(a) \ge 0$

$\therefore \forall x > a$, we have $h(x) > h(a) \ge 0$

i.e. $g(x) - f(x) > 0$

i.e. $g(x) > f(x)\ \forall x \in [a,\infty)$

Hence option (3) is correct.

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54. **Explanation:** As we know that, under a continuous function.
(1) The image of a connected set is connected
(2) The image of a compact set is compact.
Therefore
Statement (I) cannot be true as $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is compact But $(0,1)$ is not (as $(0,1)$ is not closed).
Statement (II) cannot be true as $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is compact in $\mathbb{R}$ But is not compact.
Statement (III) is true as $[0,\pi] \cup [2\pi, 3\pi]$ is compact and its image $[0,1]$ is also compact in $\mathbb{R}$.
Statement (IV) cannot true as domain $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is connected But the Range set is not connected.
Hence no option is correct.

55. **Explanation:** Surface is given by $z = 2x^2 - y^2$
$\therefore p = \frac{\partial z}{\partial x} = 4x$ and $q = \frac{\partial z}{\partial y} = -2y$
Equation of tangent plane at $(1,1,1)$ is given by
$z - 1 = \left.\frac{\partial z}{\partial x}\right|_{(1,1,1)} (x-1) + \left.\frac{\partial z}{\partial y}\right|_{(1,1,1)} (y-1)$
$z - 1 = 4(x-1) + (-2)(y-1)$
$\Rightarrow z - 1 = 4x - 4 - 2y + 2$
$i.e.\ 4x - 2y - z = 1$
Hence correct option is (4).

56. **Explanation:** Given that $f : \mathbb{R} \to \mathbb{R}$ is continuous function and
$f(x+y) = f(x) + f(y)\ \forall x, y \in \mathbb{R}$
If $y = 0$
We have, $f(x) = f(x+0) = f(x) + f(0)$
This give $f(0) = 0$ ..... (1)
Also, if $y = -x$ we have
$0 = f(0) = f(x + (-x)) = f(x) + f(-x)$
$i.e\ f(-x) = -f(x)\ \forall x \in \mathbb{R}$ ..... (2)
Also, if $x \in \mathbb{Z}$ be arbitrary,
If $x > 0$, then $f(x) = f\underbrace{(1+1+\cdots+1)}_{x times} = f(1) + f(1) + \cdots + f(1)$ (x times)
$= xf(1)$
If $x < 0$, then $f(x) = +xf(1)$
$\Rightarrow f(x) = xf(1)\ \forall x \in \mathbb{Z}$ ..... (3)
Let, $x = \frac{1}{n}$ where $n \in \mathbb{Z}$, then $f(1) = f\left[\underbrace{\frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n}}_{n\ times}\right] = n.f\left(\frac{1}{n}\right)$
$i.e.\ f\left(\frac{1}{n}\right) = \frac{1}{n} f(1)$
This prove that if $x \in \mathbb{Q}$ be arbitrary.
Then, $f(x) = xf(1)$
Let $x \in \mathbb{Q}^c$ be arbitrary.
Since f is continuous over $\mathbb{R}$, so f is continuous at x. Let $r_n \in \mathbb{Q}$ be a sequence converging to x,
Then $f(x) = \lim_{n\to\infty} f(r_n) = \lim_{n\to\infty} r_n . f(1) = xf(1)$
Hence $\forall x \in \mathbb{R}$, we have, $f(x) = xf(1)$
Clearly, if $f(1) \geq 0$, then $f(x)$ is increasing function and if $f(1) \leq 0$ then $f(x)$ is a decreasing function
Hence option (1) is correct.

57. **Explanation:** $f : \mathbb{R} \to \mathbb{R}$ is defined as $(f(x))^2 = x^2$
Also $f(x)$ is continuous function.
Then $f(x) = x \forall x \in \mathbb{R}$ is continuous and satisfies (1)
$f(x) = -x \forall x \in \mathbb{R}$ is continuous and satisfies (1)
$f(x) = [x] \forall x \in \mathbb{R}$ is continuous and satisfies (1)
$f(x) = -|x| \forall x \in \mathbb{R}$ is continuous and satisfies (1)
So option (3) is correct.

58. **Explanation:** Note that $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$ and $f(\mathbb{Q}^c) \subseteq \mathbb{Q}$
Then $f(\mathbb{R}) = f(\mathbb{Q} \cup \mathbb{Q}^c) = f(\mathbb{Q}) \cup f(\mathbb{Q}^c) \subseteq f(\mathbb{Q}) \cup \mathbb{Q}$, which is countable subset of $\mathbb{R}$.
Also $\mathbb{R}$ is connected and f is continuous, so $f(\mathbb{R})$ must be connected subset of $\mathbb{R}$.
Hence $f(\mathbb{R})$ must be countable and connected, so $f(\mathbb{R})$ must be singleton set. This Gives us that f is a constant But No constant function takes rational to irrational and irrational to rational, hence No such continuous function exist.
Hence option (4) is correct.

59. **Explanation:** Note that if $-1 < y < 1$, then,
$\frac{1}{1-y} = 1 + y + y^2 + y^3 + y^4 + \cdots\cdots$
Let $y = -x$, then,
$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots\cdots$
Integrating on both side we have,
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \cdots\cdots$
Hence Maclaurin series expansion is valid only if $x \in (-1,1)$
Hence option (3) is correct.

60. **Explanation:** Given that f is convex function, then $f''(x) \geq 0\ \forall x$

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Since $g(x) = \frac{f(x)}{x}$

Then $g'(x) = \frac{f'(x)x - f(x)}{x^2}$

$i.e., x^2 g'(x) = f'(x)x - f(x) = \phi(x)$ (say)

Then $\phi'(x) = f''(x).x + f'(x) - f'(x)$

$= xf''(x) \forall x \in (0, +\infty)$

As $x > 0$ and $f''(x) \geq 0$ for all x, we have $\phi'(x) \geq 0 \forall x$

$\Rightarrow \phi(x)$ is increasing function and $\phi(0) = 0$ as $f(0) = 0$

$\Rightarrow \phi(x) > 0 \forall x \in (0, \infty)$

$i.e., x^2 g'(x) > 0 \forall x \in (0, \infty)$

$\Rightarrow g'(x) > 0 \forall x \in (0, \infty)$

Hence $g(x)$ is increasing function for all $x \in (0, \infty)$

Correct option is (1)

61. Explanation:

Given: α, β, γ are the roots of $x^3 + ax^2 + bx + c = 0$

$\therefore \alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = b$

$\alpha\beta\gamma = -c$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= (-a)^2 - 2(b)$

$= a^2 - 2b$

$\Rightarrow$ option (1) is correct

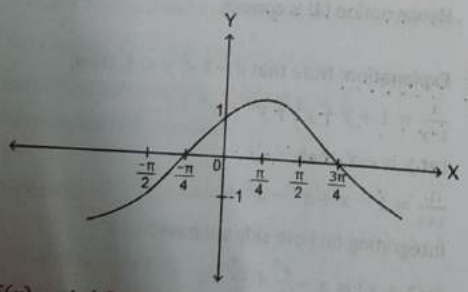
62. Explanation:

S_1 is true

Because if f is R.I then f^n is R.I but f^n is R.I then f is not R.I

$\Rightarrow$ option (1) is true

63. Explanation:



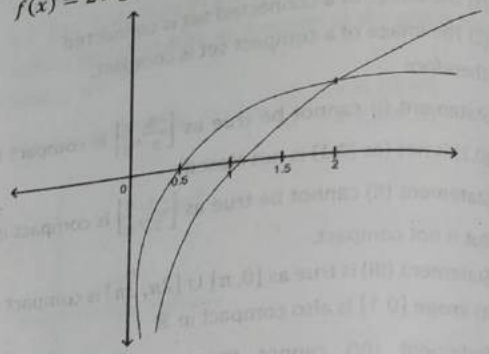
$f(x) = \sin(x) + \cos(x)$

The function is increasing form o to $\pi/4$ and decreases form $\pi/4$ to $\pi/2$

$\Rightarrow$ option (3) is correct

64. Explanation:

$f(x) = 2\log x$ and $g(x) = \log(2x)$



Option (2) is correct

65. Explanation:

$x^5 + a_1x^4 + a_2x^3 + a_3x^2 = x^2(x^3 + a_1x + a_2)$

f has at least 3 real roots

66. Explanation:

Take x=1 and y=-1

no option is correct

67. Explanation:

$f'(0) = 0$ *and* $f(x) \neq 0$ *for all* $x \in \mathbb{R}$

68. Explanation:

Given: $x^3 + x^2 + x + 1$

$\Rightarrow \alpha + \beta + \gamma = -1$

$\alpha\beta + \beta\gamma + \gamma\alpha = 1$

$\alpha\beta\gamma = -1$

$\therefore \frac{1}{\alpha-1} + \frac{1}{\beta-1} + \frac{1}{\gamma-1}$

$= \frac{(\beta-1)(\gamma-1)+(\alpha-1)(\gamma-1)+(\alpha-1)(\beta-1)}{(\alpha-1)(\beta-1)(\gamma-1)}$

$= \frac{\alpha\beta+\beta\gamma+\alpha\gamma-2(\alpha+\beta+\gamma)+3}{\alpha\beta\gamma-(\alpha\beta+\beta\gamma+\gamma\alpha)+(\alpha+\beta+\gamma)-1}$

$= \frac{1-2(-1)+3}{-1-1+(-1)-1} = -\frac{6}{4} = -\frac{3}{2}$

69. Explanation:

For each $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$.

70. Explanation:

(1) the roots of P and P' occur alternately.

71. Explanation:

The greatest integer is given by

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$[x] = 0, 0 \le x \le 1$
$1, 1 \le x \le 2$
$2, 2 \le x \le 3$
$3, 3 \le x \le 4$
$\vdots \quad \vdots$
$9, 9 \le x \le 10$
$\Rightarrow \int_0^{10}(x - [x])\,dx$
$= \int_0^{10} x\,dx - \int_0^{10}[x]\,dx$
$= \int_0^{10} x\,dx - \left\{\int_0^1 0\,dx + \int_1^2 1\,dx + \int_2^3 2\,dx + \cdots + \int_9^{10} 9\,dx\right\}$
$= \left[\frac{x^2}{2}\right]_0^{10} - \{[x]_1^2 + 2[x]_2^3 + 3[x]_3^4 + \cdots + 9[x]_9^{10}\}$
$= \frac{100}{2} - \{(2-1) + 2(3-2) + 3(4-3) + 4(5-4) + 5(6-5) + 5(7-6) + 7(8-7) + \quad 8(9-8) + 9(10-9)\}$
$= 50 - \{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9\}$
$= 50 - 45$
$= 5$
$\Rightarrow$ option (4) is correct

72. Explanation:

For the function to be continuous we have
$\lim_{x\to\alpha} -x + 3 = \lim_{x\to\alpha} x^2 - 6x + 9$
$\Rightarrow -\alpha + 3 = \alpha^2 - 6\alpha + 9$
$\Rightarrow \alpha^2 - 5\alpha + 6 = 0$
$\Rightarrow (\alpha - 2)(\alpha - 3) = 0$
$\Rightarrow \alpha = 2,3$
$\therefore$ The function f is continuous for $\{2,3\}$

73. Explanation:

Let $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \cap [0,1] \\ 0, & x \notin \mathbb{Q} \cap [0,1] \end{cases}$
As the no. of discontinuities is countable
$\therefore$ It is not Riemann integrable
$g(x) = \begin{cases} 0, & x \in \mathbb{Q} \cap [0,1] \\ 1, & x \notin \mathbb{Q} \cap [0,1] \end{cases}$
As the no. of discontinuous are uncountable
$\therefore$ The function is Riemann integrable
$\Rightarrow$ option (3) is correct

74. Explanation:

(4) does not exist

75. Explanation:

(4) Both S_1 and S_2 are FALSE

76. Explanation:

A) $a = b + c - d$

77. Explanation:

F(x) is continuous function
Also increasing
So It has maximum value at end point
F(2)= 4
Option 3 correct

78. Explanation:

$f: \mathbb{R} \to \mathbb{R}$ and $f(x) = e^x$ for one-one:
Let x_1 and x_2 be any two elements in the domain $(\mathbb{R})$ such that $f(x_1) = (x_2)$
$\Rightarrow e^{x_1} = e^{x_2}$
$\Rightarrow x_1 = x_2$
$\therefore$ f is one-one function
For onto:
We know that range of e^x is
$(0, \infty) = \mathbb{R}^+$
$\Rightarrow$ co-domain $= \mathbb{R}$
$\Rightarrow$ f is not onto function
$\Rightarrow$ option (2) is correct

79. Explanation:

$P(x) = x^4 - x$
$\Rightarrow x^4 - x = 0$
$\Rightarrow x(x^3 - 1) = 0$
$\Rightarrow x = 0, 1, w, w^2$
$\therefore$ The set of all real roots of the polynomial $P(x)$ is a set having four elements
$\Rightarrow$ option (3) is correct

80. Explanation:

Given: $f, g : \mathbb{R} \to \mathbb{R}$
$\deg(f) = 5$ and $\deg(g) = 7$
Here, $A = \{x \in \mathbb{R} : f(x) = g(x)\}$
$\Rightarrow f(x) - g(x) = 0$
$\Rightarrow \{f(x) - g(x)\}$ is a polynomial of deg(7)
$\Rightarrow |A| \le 7$
$\Rightarrow$ option (2) is correct

81. Explanation:

$10x^2 - 29x + (k + 4) = 0$
Given, one of the root is reciprocal of the other root
i.e. α and $\frac{1}{\alpha}$

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$\therefore$ product of roots $= \frac{c}{a}$

i.e. $\alpha\frac{1}{\alpha} = \frac{k+4}{10}$

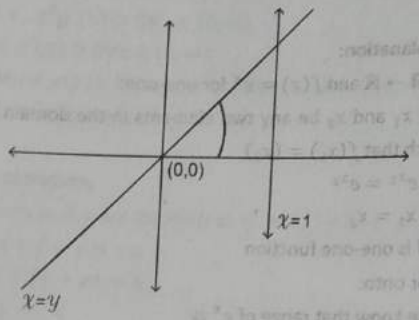
$\Rightarrow k + 4 = 10$

$\Rightarrow k = 10 - 4 = 6$

$\Rightarrow$ option (3) is correct

82. Explanation:

$I = \int_0^1 \int_y^1 e^{-x^2}\, dx\, dy$



By changing the region of integration

$I = \int_0^1 \int_0^x e^{-x^2}\, dy\, dx$

$= \int_0^1 e^{-x^2}[y]_0^x\, dx$

$= \int_0^1 x\, e^{-x^2}\, dx$

Let $x^2 = t$

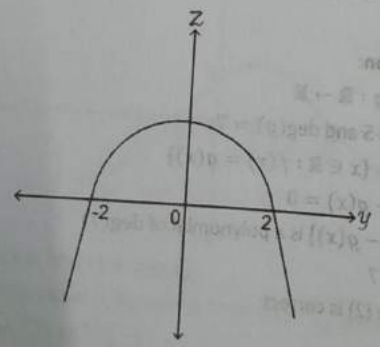
$x dx = \frac{dt}{2}$

$= \int_0^1 e^{-t}\frac{dt}{2} = \frac{1}{2}[-e^{-t}]_0^1$

$\frac{1}{2}\left[-\frac{1}{e} + 1\right] = \frac{e-1}{2e}$

83. Explanation:

Given $z = 4 - y^2$



$I = \int_{x=0}^{3}\int_{y=0}^{2}\int_{z=0}^{4-y^2} dx\, dy\, dz$

$= \int_0^3 \int_0^2 [z]_0^{4-y^2}\, dy\, dx$

$= \int_0^3 \int_0^2 [4 - y^2]\, dy\, dx$

$= \int_0^3 \left[4y - \frac{y^3}{3}\right]_0^2 dx$

$= \int_0^3 \left[8 - \frac{8}{3}\right] dx$

$= \frac{16}{3}[x]_0^3 = \frac{16}{3} \times 3 = 16$

84. Explanation:

Let $I = \int_0^\infty \frac{dx}{1+e^{rx}}$

Let $1 + e^{rx} = t$

$re^{rx}dx = dt$

$dx = \frac{dt}{re^{rx}} = \frac{dt}{r(t-1)}$

$x = 0 \Rightarrow t = 2$ and $x = \infty \Rightarrow t = \infty$

$I = \frac{1}{r}\int_2^\infty \frac{dt}{(t-1)t}$

$= \frac{1}{r}\int_2^\infty \left(\frac{1}{t-1} - \frac{1}{t}\right) dt$

$= \frac{1}{r}[\log(t-1) - \log t]_2^\infty$

$= \frac{1}{r}\left[\log\left(\frac{t-1}{t}\right)\right]_2^\infty$

$= \frac{1}{r}\left[0 - \log\frac{1}{2}\right]$

$= \frac{1}{r}[-(\log 1 - \log 2)]$

$= \frac{\log 2}{r}$

85. Explanation:

Given polynomial is $3x^5 + 9x + 7$

i.e. $P(x) = 3x^5 + 9x + 7$

$\therefore$ No. of positive real root = 0

$p(-x) = -3x^5 - 9x + 7$

$\therefore$ No. of negative real root = 1

$\Rightarrow$ option (1) is correct

86. Explanation:

Given $F(x) = e^x - x\,;\, x \in \left[-1, \frac{1}{2}\right]$

And $F'(x) = e^x - 1$

Set $F'(x) = 0 \Rightarrow e^x - 1 = 0$

$\Rightarrow e^x = 1$

$\Rightarrow x = 0$

$\therefore$ The points are $x = 0, x = -1, x = \frac{1}{2}$

$\therefore F(0) = 1, F$

$= 1 + \frac{1}{e}$

$F\left(\frac{1}{2}\right) = e^{1/2}$

$\therefore$ The maxim

87. Explanation:

$I = \int_0^{\frac{\log 3}{2}} \frac{e^{-x}}{}$

$= \int_0^{\frac{\log 3}{2}} \frac{e^x}{e^{2x}+}$

Let $e^x = t$

$e^x dx = dt$

$\therefore$ at $x = 0 \Rightarrow$

And $x = \frac{\log 3}{2}$

$\therefore I = \int_1^{\sqrt{3}} \frac{1}{t^2+1}$

$= [\tan^{-1} t]_1^{\sqrt{3}}$

$= \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$

88. Explanation:

gof is a bijecti

$\Rightarrow$ F is one-on

i.e. F cannot b

$\Rightarrow$ F and g are

$\Rightarrow$ option (A),

D) goF is on

F is one-one a

And g is onto

$\Rightarrow$ option (4) i

89. Explanation:

$f : \mathbb{R} \to \mathbb{R}$ de

$f(nx) = 2nf$

(1) clearly $f(n$

(2) $x, y \in \mathbb{R}$

$f(n(x+y))$

$f(ny)$ & be a

$f(n(nx)) = f$

$(f(nx))$

$\Rightarrow f$ is Homom

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$\therefore F(0) = 1, F(-1) = e^{-1} - (-1)$

$= 1 + \frac{1}{e}$

$F\left(\frac{1}{2}\right) = e^{1/2} - \frac{1}{2} = \sqrt{e} - \frac{1}{2} > 1$

$\therefore$ The maximum value is $1 + \frac{1}{e}$ and minimum value is 1

87. Explanation:

$$I = \int_0^{\frac{\log 3}{2}} \frac{dx}{e^{-x} + e^{x}}$$

$$= \int_0^{\frac{\log 3}{2}} \frac{e^{x}}{e^{2x} + 1} dx$$

Let $e^x = t$

$e^x dx = dt$

$\therefore$ at $x = 0 \Rightarrow t = 1$

And $x = \frac{\log 3}{2} \Rightarrow t = 3^{1/2}$

$$\therefore I = \int_1^{\sqrt{3}} \frac{1}{t^2+1} dt$$

$$= [\tan^{-1} t]_1^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

88. Explanation:

gof is a bijection

$\Rightarrow$ F is one-one and g is onto.

i.e. F cannot be onto and g cannot be one-one

$\Rightarrow$ F and g are not bijection

$\Rightarrow$ option (A), (B) and (C) are incorrect

D) goF is onto $\Rightarrow$

F is one-one and g is onto

And g is onto $\Rightarrow |Y| \geq |Z|$

$\Rightarrow$ option (4) is correct

89. Explanation:

$f : \mathbb{R} \to \mathbb{R}$ defined by

$f(nx) = 2nf(x)\ \forall n \in \mathbb{N}. V\ x \in \mathbb{R}$

(1) clearly $f(nx)$ is continuous

(2) $x, y \in \mathbb{R}$

$f(n(x+y)) = 2n(x+y) = 2n(x) + 2ny = f(nx) + f(ny)$ & be a scalar

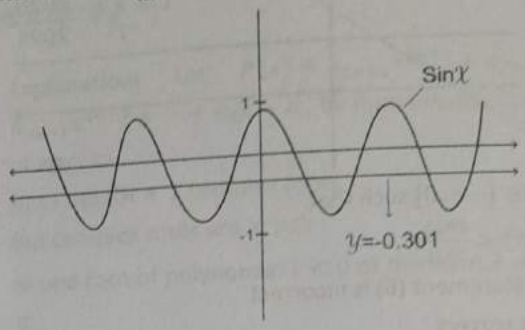
$f(n(nx)) = f(n \propto x) = f((n \propto)x) = 2n \propto (x) = \propto (f(nx))$

$\Rightarrow f$ is Homomorphism

90. Explanation:

(1) $e^{\sin 2u} = \frac{1}{2}$

$\sin 2x = \log\left(\frac{1}{2}\right) = -0.301$

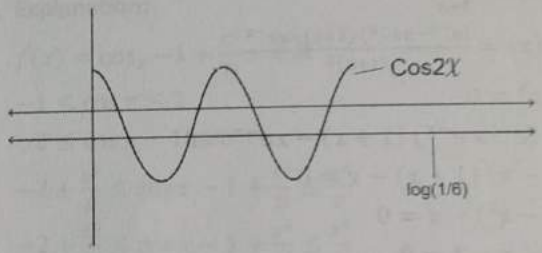


$\therefore \nexists$ infinitely many $x < 0$ such that

$e^{\sin 2x} = \frac{1}{2}$

$e^{\cos 2x} = \frac{1}{6}$

$\cos 2x = \log\left(\frac{1}{6}\right)$

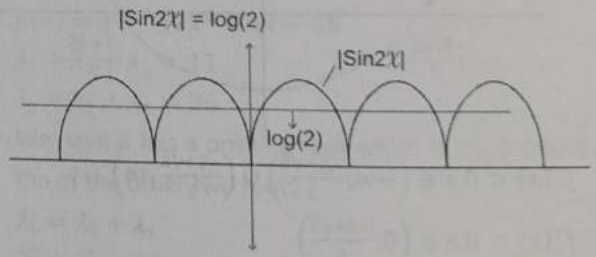


$\Rightarrow \nexists$ infinitely many $x > 0$ such that

$e^{\cos 2x} = \frac{1}{6}$

So statement (2) correct

(3) $r^{|\sin 2x|} = 2$



$\nexists$ infinitely many $x \in \mathbb{R}$ such that

$e^{|\sin x|} = 2$.

So (3) is correct

$\Rightarrow$ (1) (2) (3) are correct

91. Explanation:

(1) $f : [1, -1] \to \mathbb{R}$ be a non – constant function

$f(0) = 1$

$f(x)$ is continuous at $x = 0$

Let

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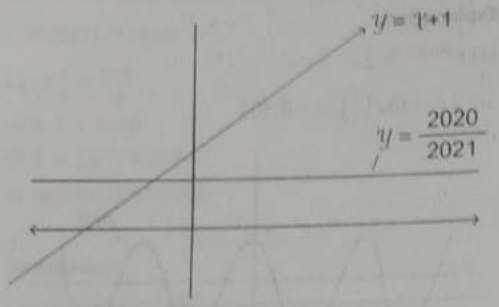


$\exists \in \in [-\epsilon, 0]$ such that

$f(x) < \frac{2020}{2021}$

So statement (B) is incorrect

A is correct

92. Explanation:

Let

$f : (0, \infty) \to \mathbb{R}$ defined by

$f(x) = \frac{x e^{-x}}{1+x}$

$f'(x) = \frac{(e^{-x} - xe^{-x})(1+x) - xe^{-x}}{(1+x)^2}$

$p(x) = 0$

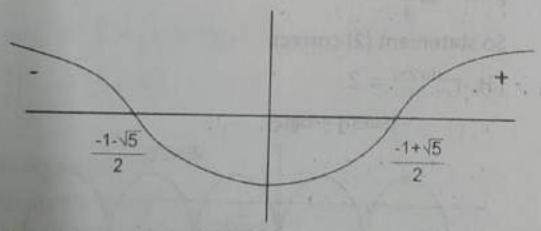
$\Rightarrow (e^{-x} x e^{-x})(1+x) - xe^{-x} = 0$

$(1-x)(1+x) - x = 0$

$(1-x^2) - x = 0$

$x^2 + x - 1 = 0$

$x = \frac{-1+\sqrt{5}}{2}$



$f'(x) < 0 \; x \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

$f'(x) > 0 \; x \in \left(0, \frac{-1+\sqrt{5}}{2}\right)$

So option (1) & (2) are wrong

$f(0) = 0$

Here we get a point where function has maximum let $i+$ be x_0

$\Rightarrow f''(x-0) < 0$

$\exists\, x_o \in (0, \infty)\, f(x_o) > 0$

$\Rightarrow f(x_0) p''(x_0) < 0$

So option (2) correct

93. Explanation:

$f_a : \mathbb{R} \to \mathbb{R}$

$f_a(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$

Let $x = (0\ 1\ 0\ 0)$

$fa(x) = |x|$

Which is not differentiable

& $fa(x)$ is polynomial in modulus

So c *is* continuous

$\Rightarrow$ statement (1) correct & (2) incorrect

94. Explanation:

$f(x, y) = x^2 + y^2 x \geq 0, y \geq 0, 2x + y \leq 1$

$L(x, y) = x^2 + y^2 + > (2x + y - 1)$

$L(x, y) = x^2 + y^2 + > (2x + y - 1)$

$L_x(x, y) = 2x + 2 > 0$

$L_y(x, y) = 2y + A = 0$

$L(x, y) = 2x + y - 1 = 0$

$x + \lambda = 0\ 2y + \lambda = 0\ 2x + y - 1 = 0$

$-\lambda = x\ y = \frac{-\lambda}{2}$

$2(-\lambda) - \frac{\lambda}{2} - 1 = 0$

$-2\lambda - \frac{\lambda}{2} - 1 = 0$

$\frac{-5\lambda}{2} - 1 = 0$

$\lambda = \frac{-2}{5} \Rightarrow x = \frac{2}{5}, y = \frac{1}{5}$

$L(x, y) = x^2 + y^2 - \frac{2}{5}(2x + y - 1)$

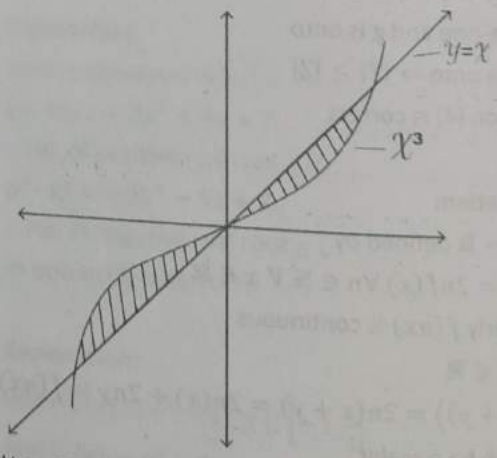
$L_{xx}(x, y) = 2 > 0$

$\Rightarrow$ minima at $\left(\frac{2}{t}, \frac{1}{t}\right)$

95. Explanation:

We have

$y = x^3\ x = y$



Here $0 < x < 1 \qquad x^3 < y \leq x$

$A = \int_0^1 \int_{x^3}^{x} 1 dy dx = \int_0^1 (x - x^3) dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$

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96. Explanation:

$f(x) = \int_0^{3x^2} t \sin t dt$

$f'(x) = 3x^2 \sin(3x^2)6x = 18x^3 \sin(3x^2)$

97. Explanation:

$\int_0^1 \int_0^1 2x^3 e^{x^2y} dx\, dy = \int_0^1 \int_0^1 2x^3 e^{x^2y} dy dx$

$= \int_0^1 [2x^3] \left[\frac{e^{x^2y}}{x^2}\right]_0^1 dx$

$= \int_0^1 2x \left[\frac{ex^2y}{}\right]_0^1 dx$

$= \int_0^1 2x[e^{x^2} - 1]\, dx$

$\int_0^1 \int_0^1 2x^3 e^{x^2y} x dy = \int_0^1 2x\, e^{x^2} dx + \int_0^1 2x dx$

Put $x^2 = 1$

$2x dx = dt$

$\int_0^1 \int_0^1 2x^3 e^{x^2y} dx dy = \int_0^1 e^t dt + \int_0^1 2x dx = e - 2$

98. Explanation:

$df, g: \mathbb{R} - \mathbb{R}$

$f(x) = e^{2x} g$ is poly of degree $n \geq 2$

(A) Let

$S = \{x \in \mathbb{R} \,|fog(x) = 1\}$

$|S| \leq n.$

$f(g(x)) = e^{2g(x)} = 1$

$f(g(x)) = 1$

$\Rightarrow 2g(x) = 0$

$g(x) = 0$

$\Rightarrow x \in s$ such that $g(x) = 0$

$|S| \leq n$

A true.

(B) $g(0) = 1 \quad g(1) = 5$

Let. e^{4x^2+1}

$9(x) = 4x^2 + 1$

$g(o) = 1 \quad g(1) = 5$

$fog(x) = e^{2(4x^2+1)} = e^{8x^2+1} + e^g \; \forall x \in (01)$

$\Rightarrow (B)$ statement wrong

(C) $S = \{x \in \mathbb{R} | fog(x) = 1\}$

$S = \{x \in \mathbb{R} \,|e^{2g(x)} = 1\}$

$g(x) = x^2 + 1$

$S = \{x \in \mathbb{R}| \, e^{2(x^2+1)} = 1\}$

$\because y_n$ is monoton but not convergent

99. Explanation: $P(x) = x^4 - 4x^3 + 2x^2 + ax + b$

$\lambda \,\&\, \frac{1}{\lambda}$ are roots of $p(x)$

$\Rightarrow P(\lambda) = \lambda^4 - 4\lambda^3 + 2\lambda^2 + a\lambda + b = 0$

$P\left(\frac{1}{\lambda}\right) = \frac{1}{\lambda^4} - \frac{4}{\lambda^3} + \frac{2}{\lambda^2} + \frac{a}{\lambda} + b = 0$

$\Rightarrow 1 - 4\lambda + 2\lambda^2 + a\lambda^3 + b\lambda^4 = 0$

$\lambda^4 - 4\lambda^3 + 2\lambda^2 + a\lambda + b = b\lambda^4 + a\lambda + 2\lambda^2 - 4\lambda + 1$

$\Rightarrow a = -4\; b = 1$

100. Explanation: Let $P(x) = a_{2n+1}x^{2n+1} + a_{2n}x^{an} + a_{2n-1}x^{2n-1} + \cdots + a_1 x + a_0$ by fundamental theorem of algebra

$p(x)$ has $2n + 1$ complex roots

But complex roots are in pair

$\Rightarrow$ one root of polynomial $i < 0$ of the form $\bar{\lambda} = \bar{z}\; z \in \not\subset$

$\Rightarrow z$ is real

$\Rightarrow p(x)$ has a real root

(b) $p(x) = x^4 + 1$

This polynomial cannot be factorized

101. Explanation:

$f(x) = \cos_x -1 + \frac{x^2}{2!} x \in \mathbb{R}$

$-1 \leq \cos x \leq 1$

$-2 \leq \cos x - 1 \leq 0$

$-2 + \frac{x^2}{2!} \leq \cos x - 1 + \frac{x^2}{2!} \leq \frac{x^2}{2!}$

$-2 + \frac{x^2}{2!} \leq \cos x - 1 + \frac{x^2}{2!} \leq \frac{x^2}{2!}$

$= -2 + \frac{x^2}{2!} \leq f(x) \leq \frac{x^2}{2!}$

$f(x) \geq 0 \; \forall x \in \mathbb{R}$

102. Explanation:

$p(x) = x^3 - 11x^2 + ax - 36$

$\lambda_1 + \lambda_2 + \lambda_3 = 11$

$\lambda_1 + \lambda_2 + \lambda_3 = 36$

We have it has a positive root which is the product of the of the other two roots

$\lambda_1 = \lambda_2 + \lambda_3$

$\lambda_2^2 + \lambda_3^2 = 36$

$(\lambda_2 + \lambda_3)^2 = 36$

$\lambda_2 + \lambda_3 = 36$

$\Longrightarrow \lambda_1 = 6$

$\lambda_2 + \lambda_3 = 6$

$\lambda_3 = \frac{6}{\lambda_2}$

$6 + \lambda_2 + \frac{6}{\lambda_2} = 11$

$\lambda_2 + \frac{6}{\lambda_2} = -5$

$\lambda_2^2 + 6 = -5\lambda_2$

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$\lambda_2^2 + 5\lambda_2 + 6 = 0$

$\frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2}$

$\lambda_2 = 3 \text{ or } 2$

If $\lambda_2 = 3 \Rightarrow \lambda_3 = 2$

$\lambda_2 = 3 \Rightarrow \lambda_3 = 3$

$\Rightarrow$ roots of Polynomial, 6, 2, 3

$P(2) = 0$

$\Rightarrow 8 - 11 \times 4 + 2a - 36 = 0$

$8 - 44 + 2a - 36 = 0$

$-36 + 2a - 36 =$

$a = 36$

103. Explanation:

$$\lim_{x\to\infty}\left(\frac{sinx}{2}\right)^{\frac{1}{x^2}} = e^{\lim_{n-2x^2}\frac{1}{2}\left[\frac{sinx}{x}-1\right]}$$

$$= e^{\lim_{x\to 0}\frac{sinx-x}{x^3}}$$

$$= e^{\lim_{x\to 0}\frac{cosx-1}{3x^2}}$$

$$= e^{\lim_{x\to 0}\frac{-sinx-1}{6x}}$$

$$= e^{\frac{-1}{6}}$$

104. Explanation:

$\frac{\pi}{4}\frac{T}{4}$

$$\lim_{n\to\infty}\frac{1}{n^2}\sum_{K=1}^{n}\sqrt{n^2-k^2}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{K=1}^{n}\sqrt{1-\left(\frac{K}{n}\right)^2}$$

$$= \int_0^1\sqrt{1-x^2}$$

where $x = \frac{K}{n}$

$$= \frac{\pi}{4}$$

105. Explanation:

$$\lim_{x\to 0}(1-\sin x\cos x)^{cosec\ 2x}$$

1^{∞}form

$$e^{\lim_{x\to 0} cosec\ 2x(1-\sin x\cos x)}$$

$$e^{\lim_{x\to 0}\frac{\sin x\cos x}{\sin 2x}=e^{-\frac{1}{2}}}$$

106. Explanation:

$$\lim_{n\to\infty}\frac{1}{n^6}\sum_{k=1}^{n}k^5 = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^5$$

$$= \int_0^1 x^5 dx$$

$$= \frac{1}{6}$$

107. Explanation:

$$\lim_{x\to\infty} x\left(\log\left(1+\frac{x}{2}\right)-\log\left(\frac{x}{2}\right)\right)$$

$$y = \lim_{x\to\infty} x\left(\log\left(\frac{1+\frac{x}{2}}{\frac{x}{2}}\right)\right)$$

$$y = \lim_{x\to\infty}\log\left(\frac{2}{x}+1\right)^x$$

$$y = \lim_{x\to\infty}\log\left(\frac{2}{x}+1\right)^x$$

$$e^y = \lim_{n\to\infty}\left(\frac{2}{x}+1\right)^x$$

$$e^y = e^{\lim_{n\to\infty} x\left(\frac{2}{x}+1-1\right)}$$

$$= e^2$$

108. Explanation: $\int_0^{\frac{\pi}{2}}\log\tan\theta\ d\theta$

$$\int_b^a F(x)dx = \int_a^b F(a+b-2)dx$$

$$\int_0^{\frac{\pi}{2}}\log\left(\tan\left(\frac{\pi}{2}-\theta\right)\right)d\theta = \int_0^{\frac{\pi}{2}}\log\tan\theta\ d\theta$$

$$\int_0^{\frac{\pi}{2}}\log(\cos t\theta)\,d\theta = \int_0^{\frac{\pi}{2}}\log(\tan\theta)\ d\theta$$

$$\int_0^{\frac{\pi}{2}}\log\left(\frac{1}{\tan\theta}\right)d\theta = \int_0^{\frac{\pi}{2}}\log(\tan\theta)\ d\theta$$

$$-\int_0^{\frac{\pi}{2}}\log(\tan\theta)\,d\theta = \int_0^{\frac{\pi}{2}}\log(\tan\theta)\ d\theta$$

$$\Rightarrow \int_0^{\frac{\pi}{2}}\log(\tan\theta)\,d\theta = 0$$

109. Explanation:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}p\left(\frac{k}{n}\right) = \int_0^1 F'(x) = F(1)-F(0)$$

110. Explanation:

$$y = \lim_{x\to 0}(1+3x^2)^{5\cot x+2\left(\frac{cosecx}{x}\right)}$$

1^{∞} for m

$$y = e^{\lim_{x\to 0}5\cot+2\left(\frac{cosecx}{x}\right)[1+3x^2-1]}$$

$$= e^{\lim_{x\to 0}\left[5\cot x+2\left(\frac{cosecx}{x}\right)\right][1+3x^2-1]}$$

$$= e^{\lim_{x\to 0}[15x^2\cot x+6xcosecx]}$$

$$\lim_{x}\left[15x^2\left(\frac{\cos x}{\sin x}\right)+\frac{6x}{\sin x}\right]$$

$$\lim_{x\to 0}\left[\frac{15x^2\cos x+6x}{\sin x}\right]$$

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$$\lim_{x\to 0}\left[\frac{15[-x^2\sin x+2x<\cos x]+6}{\cos x}\right]=6$$

$y=e^6$

111. Explanation:

1) since $|\sin x|\le x$

$|\sin x-\sin y|\le|x-y|\forall\, x,y\in IR$

$\Rightarrow$ Statement a correct

2) $|\sin 2x-\sin 2y|\le 2|x-y|\forall\, x,y\in IR$

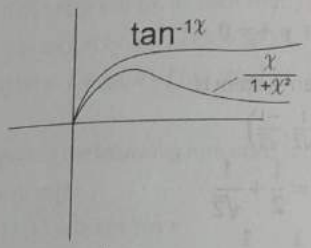
$\Rightarrow$ Statement b incorrect

3) $|\sin^2 x-\sin^2 y|\le|x^2-y^2|\ \forall\ x,y\in IR$

Statement C incorrect

112. Explanation:

1. $f(x)\tan^{-1}x$ & $g(x)\frac{x}{1+x^2}$



$f(0)=\tan^{-1}(0)=0$ & $g(0)=0$

$f(1)=\tan^{-1}(1)=\frac{\pi}{4}$ $\quad g(1)=\frac{1}{2}$

$\lim_{x\to\infty}\tan^{-1}(x)=\infty$ & $\lim_{x\to\infty}\frac{x}{1+x^2}=0$

$\tan^{-1}x>\frac{x}{1+x^2}$

2. $f(x)=\log(1+x)$ > $g(x)=\frac{x}{1+x}$

113. Explanation:

(1) $a_0+\frac{a_1}{2}+\frac{a_2}{3}+\cdots+\frac{a_n}{n+1}=0$

$P(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n$

$F(1)=\int P(1)=a_0+\frac{a_1}{2}+\frac{a_2}{3}+\cdots+\frac{a_n}{n+1}=0$

$F(0)=\int P(0)dx=0$

$F(1)=F(0)$

Then by rolls theorem

$\exists\, C\in(0\ 1)$

Such that $F(c)=0$

$\Rightarrow P(C)=0$

$\Rightarrow P(x)$ has root in (0,1)

(2) True

$F.[a,b]\to\mathbb{R}$

$\frac{F(a)}{a}=\frac{F(b)}{b}$

Let $g(x)=\frac{F(x)}{x}$

$g(a)=\frac{F(a)}{a}$

$g(b)=\frac{F(b)}{b}$

$g(a)=g(b)$

$\exists x_0\in(a,b)$ such that $g'(x_0)=0$

$\frac{xF'(x)-F(x)}{x^2}=0$

$\Rightarrow F(x_0)=x_0F'(x_0)$

(3) $x^{13}+7x^3-5=p(x)$

Only one sing chang in $P(x)$

$\Rightarrow P(x)$ can have at least one positive root

$P'(x)=13x^{12}+21x^2\ge 0$

$\Rightarrow P(x)$ increasing function and P(x) is odd degree polynomial

$\Rightarrow F(x)$ has only one real root

114. Explanation:

$\int_0^1\max\left\{x,\frac{1}{2}-x\right\}dx$

$$\int_0^{1/4}\left(\frac{1}{2}-x\right)dx+\int_{1/4}^{1}xdx$$

Since

$$F(x)=\max\left\{x,\frac{1}{2}-x\right\}$$

$$=\begin{cases}\frac{1}{2}-x & 0\le x<\frac{1}{4}\\ x & \frac{1}{4}\le x<1\end{cases}$$

$$=\left[\frac{x}{2}-\frac{x^2}{2}\right]_0^{1/4}+\left[\frac{x^2}{2}\right]_{1/4}^{1}$$

$$=\frac{1}{8}-\frac{1}{32}+\left[\frac{1}{2}-\frac{1}{3^2}\right]$$

$$=\frac{5}{8}-\frac{2}{32}$$

$$=\frac{5}{8}-\frac{1}{16}$$

$\int_0^1\max\left\{x,\frac{1}{2}-x\right\}dx=\frac{9}{16}$

115. Explanation:

$\lim_{\theta\to 0}(1-2\tan\theta)^{cat\theta}$

1^∞ form

$\lim_{\theta\to 0}(1-2\tan\theta-1)\cot\theta$

$\lim_{\theta\to 0}(-2\tan\theta)\cot\theta$

e^2

$\Rightarrow\lim_{\theta\to 0}(1-2\tan\theta)^{\cot\theta}=e^{-2}$

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116. Explanation:

$\lim_{n\to 0}\frac{1}{n}\int_{t-4}^{t+h} f(s)\,ds$

$\lim_{n\to 0}\left[\frac{\int_{t-h}^{t+h} f(s)ds}{n}\right]$

$\left(\frac{0}{0}\right)$ form

$\lim_{n\to 0}\left[\frac{f(t+n)+f(t-n)}{1}\right]$

$\lim_{n\to 0}\big(f(t+n)+f(t-n)\big)$

$= 2f(t)$

117. Explanation: $x - 10x^3 + 26x^2 - 10x + 1$

$(x^2+ax+1)(x^2+bx+1)=0$

$x^4+bx^3+x^2+ax^3+abx^2+ax+x^2+bx+1 = 0$

$x^4+(a+b)x^3+(2+ab)x^2+(a,b)x+1$

$\Rightarrow a+b=-10$

$2+ab=26$

$ab=24$

$a=\frac{24}{b}$

$a+b=-10$

$\frac{14}{b}+b=-10$

$24+10b+b^2=0$

$a=-6 \quad b=-4$

$x^4 \pm 10x^3+26x^2-10x+1=(x^2-6x+1)(x^2-4x+1)$

$x^2-6x+1=0$ or $x^2-4x+1=0$

Roots are

$3\pm 2\sqrt{2}, \qquad 2\pm\sqrt{3}$

118. Explanation: $\lim_{x\to 0}(cosx)^{\frac{1}{\sin^2 x}}$

1 form

$\lim_{e^{x\to 0}}\frac{1}{\sin^2 x}(\cos x-1)$

e^{+} form

$\lim_{e^{x\to 0}}\frac{-\sin x}{2\sin x\cos x}$

$e^{-\frac{1}{2}}$

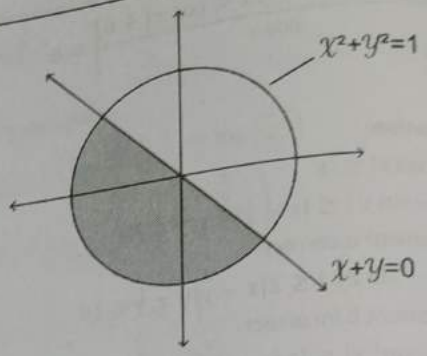
119. Explanation:

$S=\{(x,y)|x^2+y^2\le 1, x+y\le 0\}$ $\qquad F(x,y)$

$= x^2+y^2$

$L(x,y)=x^2+y^2+\lambda(x^2+y^2-1)+\mu(x+y)$

Find all partial derivation



$\frac{\partial L}{\partial x}=0 \Rightarrow 2\lambda x+\mu+1=0$

$\frac{\partial L}{\partial y}=0 \Rightarrow 2\lambda y+\mu+1=0$

$\frac{\partial L}{\partial \lambda}=0 \Rightarrow x^2+y^2+=0$

$\frac{\partial L}{\partial \mu}=0 \Rightarrow x+y+=0$

Solution of given system is

$\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ & $\left(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$

$F\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)=\frac{1}{2}+\frac{1}{\sqrt{2}}$

$F\left(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)=\frac{1}{2}-\frac{1}{\sqrt{2}}$

120. Explanation:

F is continuous Function from the real line IR to close interval [1,3]

1..$F^{-1}(1)$ & $F^{-1}(3)$ are singleton set

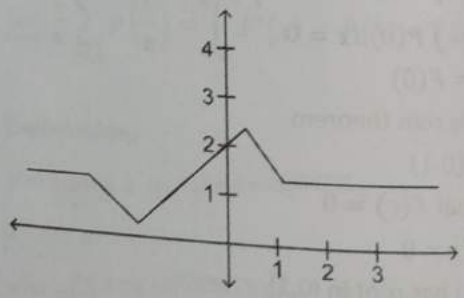
$F^{-1}(x)$ consider of exactly two real numbers

For $x\in(1,2)U(2,3)$

$F: IR\to[1,3]$

FROM THIS GRAPH WE CAN SEE

$|F^{-1}(2)|$ IS UNCOUNTABLY INFINITE.



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Unit 1.5: Continuity & Uniform Continuity

[CU-CET]

[CUCET 2016]

1. Define the function $f : R \to R$ by
$f(x) = \{sin\ sin\ |x|, if\ x\ is\ rational\ 0, otherwise$
Then f is continuous
(1) at all rational points
(2) at all irrational points
(3) at all $x = k\pi$, where k is any integer
(4) at all $x \neq k\pi$, where k is any integer

[CUCET 2016]

2. Let f and $g[a, b] \to R$ be two continuous functions such that $f(a) < g(a)$and $f(b) > g(b)$. Then
(1) there is a $c \in (a, b)$ such that $f(c) + g(c) = 0$
(2) there is a $c \in (a, b)$ such that $f(c) - g(c) = 0$
(3) for all $c \in (a, b). f(x) = g(x)$
(4) for all $c \in (a, b). f(x) \neq g(x)$

[CUCET 2016]

3. Which of the following functions is uniformly continuous on $(0, \infty]$?
(1) $f(x) = x\ sin\ sin\ x$
(2) $g(x) = sinx^2$
(3) $h(x) = e^x$
(4) $k(x) = sin\ sin\ (sin\ sin\ x\ )$

[CUCET 2019]

4. If $f: R \to R$ is defined by $f(x) = |x - c|$, for all $x \in R$; then
(1) f is discontinuous
(2) f is differentiable
(3) f is continuous but not differentiable
(4) f is continuously differentiable

[CUCET 2019]

5. The function $f(x) = \{x\ sin\ sin\ 1/x,\ when\ x \neq 0\ 0,$ $when\ x = 0$ is
(1) continuous at $x = 0$
(2) derivable at $x = 0$
(3) discontinuous at $x = 0$
(4) infinitely differentiable at $x = 0$

[CUCET 2020]

6. Let $f: R \to R$ be a continuous function satisfying $x + \int_0^x f(t)dt = e^{x-1}$, for all $x \in R$. Then the set $\{x \in R: 1 < f(x) \leq 2\}$ is the interval
(1) $[log\ 2, log\ 3]$
(2) $[2log\ 2, 3log\ 3]$
(3) $[e - 1, e - 2]$
(4) $[0, e]$

Answer Key					
1	2	3	4	5	6
3	2	4	3	1	1

[PRACTICE QUESTIONS]

1. Which one of the following statements is true?
(1) The functions $\sin x$ and x^2 are uniformly continuous on $[0, \infty)$.
(2) The functions $\sin x$ and e^{-x} are uniformly continuous on $[0, \infty)$.
(3) The functions e^{-x} and $\frac{1}{x}$ are uniformly continuous on $[0, \infty)$.
(4) The functions x^2 and $\frac{1}{x}$ are uniformly continuous on $[0, \infty)$.

2. Let $f(x) = x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)\right)$, then :
(1) $\lim_{x\to 0} f(x)$ does not exist
(2) $\lim_{x\to 0} f(x) = 1$
(3) $\lim_{x\to 0} f(x) = 0$
(4) $\lim_{x\to 0} f(x) = -1$

3. Suppose the inequality $\frac{1}{2} - \frac{x^2}{24} < \frac{1-\cos x}{x^2} < \frac{1}{2}$ holds for the value of x close to zero, then :
(1) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) > \frac{1}{2}$
(2) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) < \frac{1}{2}$
(3) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) = \frac{1}{2}$
(4) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ does not exist

4. Let $f(x) = \begin{cases} x^2, x < 2 \\ 3, x = 2 \\ 2, x > 2 \end{cases}$ Then one of the following is not true.
(1) $\lim_{x\to 2} f(x) \neq 2$
(2) $\lim_{x\to 2} f(x) \neq 3$
(3) $\lim_{x\to 2} f(x) \neq 4$
(4) $\lim_{x\to 2} f(x) = 3$

5. Let $f(x) = \frac{x(x^2-1)}{|x^2-1|}$. Then
(1) f can be continuously extended at $x = 1$
(2) f can be continuously extended at $x = -1$
(3) f can be extended continuously at x = 1 and $x = -1$
(4) f can not be continuously extended at $x = -1$

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6. Let $f(x) = \sin x$ and $g(x) = x^2$, then :
   (1) both f and g are uniformly continuous on IR
   (2) f is uniformly continuous on IR but g is not uniformly continuous on IR
   (3) neither f nor g is uniformly continuous on IR
   (4) g is uniformly continuous on IR

7. Let $g_n(x) = \frac{1}{n(1+x^2)}, n \geq 1$ and $f_n(x) = \frac{x+nx}{n}, n \geq 1$, then:
   (1) $g_n \to 0$ uniformly on IR but $\{f_n\}_{n=1}^{\infty}$ does not converge uniformly on IR
   (2) both $\{f_n\}_{n=1}^{\infty}$ and $\{g_n\}_{n=1}^{\infty}$ converge uniformly on IR
   (3) neither of $\{f_n\}_{n=1}^{\infty}$ nor $\{g_n\}_{n=1}^{\infty}$ converges uniformly on IR
   (4) $f_n \to 0$ uniformly on IR

8. Let $f : IR \to IR$ be a monotonic function, then:
   (1) The set of points where f discontinuous is a finite set.
   (2) The set of points where f is discontinuous is a countable set
   (3) The set of points where f is continuous is a countable set
   (4) The set of points where f is discountinous is an uncountable set

9. If a function $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x+y) = f(x) + f(y)\ \forall x, y \in R$, then $f(x)$ is
   (1) $x^3 f(1)$ (2) $x^2 f(1)$
   (3) $xf(1)$ (4) $x^4 f(1)$

10. $\lim_{n\to\infty} 2^{-2/n}$ is
    (1) 1 (2) 2
    (3) 3 (4) 4

11. The function f defined by $f(x,y) = |x| + |y|$ is
    (1) not continuous at (0,0)
    (2) differentiable at (0,0)
    (3) continuous but not differentiable at (0,0)
    (4) continuous as well as differentiable at (0,0)

12. Which function is continuous at $x = 0$?
    (1) $\sin\left(\frac{1}{x}\right)$ (2) $\sin\left(\frac{1}{x^2}\right)$
    (3) $\tan^{-1}\left(\frac{1}{x}\right)$ (4) $\tan x$

13. Consider the sequence $\langle f_n \rangle$ defined by $f_n(x) = 1/(1+x^n)$ for $x \in [0,1]$. Let $f(x) = \lim_{n\to\infty} f_n(x)$ Then
    (1) For $0 < a < 1$, $\langle f_n \rangle$ converges uniformly to f on $[0,a]$
    (2) the sequence $\langle f_n \rangle$ converges uniformly to f on $[0,1]$
    (3) the sequence $\langle f_n \rangle$ converges uniformly to f on $[1/2,1]$
    (4) the sequence $\langle f_n \rangle$ converges uniformly to f on $[0,1]$

14. For $n \in \mathbb{N}$, let $f_n(x) = \frac{\sin x}{x} + \frac{\cos x}{\sqrt{n}}$ for $x \in (,\pi/2)$ Then
    (1) $\langle f_n \rangle$ converges uniformly on $(0,\pi/2)$ but not on $(0,\pi/2]$
    (2) $\langle f_n \rangle$ converges uniformly on $(0,\pi/2]$
    (3) $\langle f_n \rangle$ converges uniformly on $(0,\pi/4)$ but not on $(0,\pi/4]$
    (4) none of these

15. Let $f : [a,b] \to [a,b]$ be a continuous function. Then
    (1) $\lim_{n\to\infty} \int_a^b f(x) \sin nx dx = \pi$
    (2) $\lim_{n\to\infty} \int_a^b f(x) \cos nx dx = \pi$
    (3) $\lim_{n\to\infty} \int_a^b f(x) \sin nx dx = 1$
    (4) $\lim_{n\to\infty} \int_a^b f(x) \cos nx dx = 0$

16. Which one of the following sequence converges uniformly on the indicated set?
    (1) $f_n(x) = (1-|x|)^n; x \in (-1,1)$
    (2) $f_n(x) = \frac{1}{n}\sin nx; x \in \mathbb{R}$
    (3) $f_n(x) = x^n; x \in [0,1]$
    (4) $f_n(x) = \frac{1}{1+x_n}; x \in [0,\infty)$

17. Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = |x-1| + |x-2|$. Let $S_1 = \{x | f$ is comtinuos at $x\}$ and $S_2 = \{x | f$ is diffrentiable at $x\}$ then
    (1) $S_1 = \mathbb{R}, S_2 = \mathbb{R}$.
    (2) $S_1 = \mathbb{R}, S_2 = \mathbb{R} / \{1.2\}$.
    (3) $S_1 = \mathbb{R}/\{1.2\}, S_2 = \mathbb{R}$
    (4) $S_1 = \mathbb{R}/\{1,2\}, S_2 = \mathbb{R}/\{1,2\}$.

18. The value of $\lim_{x\to 0}(\cos x)^{(1/\sin^x)}$ is
    (1) $\exp(-1)$ (2) $\exp(1)$
    (3) $\exp(-1/2)$ (4) $\exp(1/2)$

19. The point of continuity of the
    by
    $f(x) = \begin{cases} |x^2-1| & \text{if } x \text{ is i} \\ 0, & \text{If is r} \end{cases}$
    (1) $x = -1, x = 0, x = 1$
    (3) $x = -1, x = 0$

20. Let $f, g : (0,1) \to \mathbb{R}$ Let $f(x$
    x^2 Then
    (1) both f and g are uniform
    (2) f is uniformly conti'uc continuous.
    (3) f is not uniformly con co'tinuous.
    (4) both f and, g are not uni

21. If $f : \mathbb{R} \to \mathbb{R}$ and $\lim_{h\to 0}(f(x+$
    0 for all $x \in \mathbb{R}$, then
    (1) f need not be continuo
    (2) f is continuous but not
    (3) f is differentiable but f
    (4) f is differentiable and f

22. The value of $\lim_{x\to 0} \frac{e^{\left(\frac{1}{x}\right)} - e^{\left(-\frac{1}{x}\right)}}{e^{\left(\frac{1}{x}\right)} + e^{\left(-\frac{1}{x}\right)}}$
    (1) 0
    (2) 1
    (3)-1
    (4) none of the above.

23. The value of $\lim_{x\to 0+}\left(1+\frac{1}{x}\right)$
    (1) 0
    (3) e^2

24. What is the value of $\lim_{x\to 2}$
    (1) - log2
    (3) 1 - 2 log2

25. A function $f : (a,b) \to$
    continuous if for every ϵ
    that $|f(x) - f(y)| < \epsilon$
    is independent of x and y
    Let $f : (0,1) \to \mathbb{R}$ be the
    the following assertions:

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19. The point of continuity of the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} |x^2 - 1| & if\ x\ is\ irrational \\ 0, & If\ is\ rational \end{cases}$ are

(1) $x = -1, x = 0, x = 1$ (2) $x = -1, x = 1$

(3) $x = -1, x = 0$ (4) $x = 0, x = 1$

20. Let $f, g: (0,1) \to \mathbb{R}$ Let $f(x) = x \sin\left(\frac{1}{x^2}\right)$ and $g(x) = x^2$ Then

(1) both f and g are uniformly continuous.

(2) f is uniformly conti'uous but g is not uniformly continuous.

(3) f is not uniformly continuous but g is uniformly co'tinuous.

(4) both f and, g are not uniformly continuous.

21. If $f: \mathbb{R} \to \mathbb{R}$ and $\lim_{h \to 0}(f(x+h) - f(x-h)) = 0$ for all $x \in \mathbb{R}$, then

(1) f need not be continuous.

(2) f is continuous but not differentiable.

(3) f is differentiable but f' need not be continuous.

(4) f is differentiable and f' is continuous.

22. The value of $\lim_{x \to 0} \frac{e^{\left(\frac{1}{x}\right)} - e^{\left(\frac{-1}{x}\right)}}{e^{\left(\frac{1}{x}\right)} + e^{\left(\frac{-1}{x}\right)}} +$

(1) 0

(2) 1

(3) -1

(4) none of the above.

23. The value of $\lim_{x \to 0+} \left(1 + \frac{1}{x}\right)^{2x}$ is

(1) 0 (2) 1

(3) e^2 (4) $+\infty$

24. What is the value of $\lim_{x \to 2} \frac{(1+x)^{\frac{1}{x}} - 2}{x - 1}$?

(1) - log2 (2) - 2 log2

(3) 1 - 2 log2 (4) 1 - 3 log2

25. A function $f: (a, b) \to \mathbb{R}$ is said to be uniformly continuous if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - f(y)| < \epsilon$ whenever $|x - y| < \delta$ (and δ is independent of x and y).

Let $f: (0,1) \to \mathbb{R}$ be the map given by f(x) = $\sqrt{x}$ consider the following assertions:

(A) f is differentiable on (0, 1).

(B) f is differentiable and f' is bounded on (0, 1).

(C) f is uniformly continuous on (0, 1)

(D) f is differentiable and f' is uniformly continuous on (0, 1).

Which of the above assertions is/are correct?

(1) A only (2) A and C only

(3) A, B and C only (4) A, B and D only

26. How many solutions does the equation $x^x = \left(\frac{1}{2}\right)^{\frac{1}{2}}$ have in the open interval (0, 1)?

(1) one

(2) two

(3) more than 2 but finitely many

(4) infinitely many

27. Let $f: \mathbb{R} \to \mathbb{R}$ be a given function. List those amongst the following properties which will ensure that f is uniformly continuous:

(1) for all x and $y \in \mathbb{R}$,

$|f(x) - f(y)| \leq |x - y|^{\frac{1}{2}}$.

(2) $f(x) = \sum_{n=1}^{\infty} \frac{g(x-n)}{2^n}$

(3) Bothe (1) & (2)

(4) None of the above

where $g: \mathbb{R} \to \mathbb{R}$ is a uniformly continuous function such that the series converges for each $x \in \mathbb{R}$.

28. Pick out the functions which are not continuous at least at one point in the real line:

(1) $f(x) = \begin{cases} 1 & if \quad x\ is\ rational. \\ 0 & if \quad x\ is\ irrational. \end{cases}$

(2) $f(x) = \begin{cases} x & if \quad x\ is\ rational. \\ 0 & if \quad x\ is\ irrational. \end{cases}$

(3) $f(x) = \begin{cases} \sin \pi x & if \quad x\ is\ rational. \\ 0 & if \quad x\ is\ irrational. \end{cases}$

(4) None of the above

29. Pick out the functions which is not uniformly continuous:

(1) $f(x) = \frac{1}{x}, x \in]0,1[.$

(2) $f(x) = \frac{\sin x}{x}, x \in]0,1[.$

(3) $f(x) = \sin^2 x, x, \in \mathbb{R}.$

(4) None of the above

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30. Let $C^1(\mathbb{R})$ denote the set of all continuously differentiable real valued functions defined on the real line. Define

$A = \left\{f \in C^1 \mid f(0) = 0, f(1) = 1, |f'(x)| \leq \frac{1}{2} \text{ for all } x \in \mathbb{R}\right\}$

where f' denotes the derivative of the function f. Pick out the true statement:

(1) A is an empty set.

(2) A is a finite and non-empty set.

(3) A is an infinite set.

(4) None of the above

31. Pick out the non-uniformly continuous functions from the following and, in such cases, given $\varepsilon > 0$, find $\delta > 0$ explicitly as a function of ε so that $|f(x) - f(y)| < \varepsilon$ whenever $|x - y| < \delta$.

(1) $f(x) = \sqrt{x}, 1 \leq x \leq 2$.

(2) $f(x) = x^3, x \in \mathbb{R}$.

(3) $f(x) = \sin^2 x, x \in \mathbb{R}$.

(4) None of the above

32. Which of the following functions are not uniformly continuous?

(1) $f(x) = \sin^2 x, x \in \mathbb{R}$.

(2) $f(x) = x \sin\frac{1}{x}, x \in]0,1[$.

(3) $f(x) = x^2, x \in \mathbb{R}$.

(4) None of the above

33. Which of the following functions are continuous?

(1) $f(x) = |x| + (x - [x])^{[x]}, x \geq \frac{1}{2}$,

where $[x]$ denotes the largest integer less than, or equal to, x.

(2) $f(x) = \lim_{n\to\infty} \frac{x^2 e^{nx} + x}{e^{nx} + 1} x \in \mathbb{R}$.

(3) $f(x) = \lim_{n\to\infty} \frac{1}{n} \log(e^n + x^n), x \geq 0$.

(4) None of the above

34. Pick out the uniformly continuous functions over the interval $]0,1[$:

(1) $f(x) = \sin\frac{1}{x}$

(2) $f(x) = x \sin\frac{1}{\sqrt{x}}$

(3) $f(x) = \exp\left(-\frac{1}{x^2}\right)$

(4) None of the above

35. Let $[x]$ denote the largest integer less than, or equal to, $x \in \mathbb{R}$. Find the discontinuous function (if any) of the following functions:

(1) $f(x) = [x^2] \sin \pi x, x > 0$;

(2) $x = \sqrt{n}$

(3) $x = n$

(4) $f(x) = [x] \sin \pi x, x < 0$;

36. Pick out the uniformly continuous functions:

(1) $f(x) = \cos x \cos\frac{\pi}{x}, x \in]0,1[$;

(2) $f(x) = \sin x \cos\frac{\pi}{x}, x \in]0,1[$;

(3) $f(x) = \sin^2 x, x \in [0, \infty[$;

(4) None of the above

37. List the sets of points of discontinuity, if any, for the following functions.

(1) $f : [-1,1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1 \text{ if } x \text{ is irrational}, \\ 0 \text{ if } x \text{ is rational}. \end{cases}$$

(2) $f : [-1,1] \to \mathbb{R}$ defined by

$$f(x) \begin{cases} x & \text{if } x \text{ is irrational}, \\ 0 & \text{if } x \text{ is rational} \end{cases}$$

(3) Bothe (1) & (2)

(4) None of the above

where $[x]$ is the largest integer less than, or equal to x and $(x) = x - [x]$.

38. Find the points where the following function $f : \mathbb{R} \to \mathbb{R}$ is continuous:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational}, \\ \sin|x| & \text{if } x \text{ is irrational}. \end{cases}$$

(1) $\alpha = \pi$

(2) $\alpha = n$

(3) $\alpha = n\pi$

(4) $\alpha \geq n\pi$

39. Define:

$$f(x) = \begin{cases} a_n + \sin \pi x \text{ if } x \in |2n, 2n+1|, \\ b_n + \cos \pi x \text{ if } x \in]2n-1, 2n[. \end{cases}$$

where n varies over $\mathbb{N} \cup \{0\}$. Find all possible sequences $\{a_n\}$ and $\{b_n\}$ such that f is continuous on $[0, \infty[$.

(1) $a_n = b_n + 1$

(2) $a_n = b_n - 1$

(3) $a = b + 1$

(4) $a_n = b + 1$

40. Which of the following functions are uniformly continuous on the prescribed domain?

(1) $f(x) = x \sin\frac{1}{x}$ on $]0,1[$.

(2) $f(x) = e^x \cos\frac{1}{x}$ on $]0,1[$.

(3) $f(x) = \sin(\sin x)$ on $]0, x[$.

(4) None of the above

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41. Find the relationship between the height (h) and the radius (r) of the right circular cylinder of minimum total surface area amongst all right circular cylinders of volume $10\ cm^3$.

(1) $h = 2r$ (2) $h = 8r$

(3) $h = 4r$ (4) $h = 6r$

42. Let $n \in \mathbb{N}$ and let $J_n = \int_0^{\frac{\pi}{2}} \cos^n x\, dx$. Which of the following statements are true?

(1) J_n is rational for all $n \geq 2$.

(2) J_n is irrational for all $n \geq 2$.

(3) J_n is rational if $n = 7$.

(4) None of the above

43. Evaluate:

$\int_0^5 \max\{x^2, 6x - 8\}dx$

(1) 43 (2) 33

(3) 31 (4) 41

44. Which of the following functions is uniformly continuous on the given domain? Choose the correct options:

(1) $1/x^2$ on $[1, \infty)$.

(2) $1/x$ on $(0, \infty)$.

(3) $x \sin x$ on the real line $\mathbb{R}$.

(4) $\tan^{-1} x$ on the real line $\mathbb{R}$.

Answer Key									
1	2	3	4	5	6	7	8	9	10
2	3	3	4	4	2	2	2	3	1
11	12	13	14	15	16	17	18	19	20
3	4	1	2	4	2	2	3	2	4
21	22	23	24	25	26	27	28	29	30
4	2	3	3	2	2	1	1	4	1
31	32	33	34	35	36	37	38	39	40
3	3	2	2	1	2	2	3	1	1
41	42	43	44						
1	3	1	3						

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EXPLANATIONS

[CU-CET]

1. **Explanation:** at all $x = k\pi$, where k is any integer

2. **Explanation:**

$f \& g : [a,b] \to \mathbb{R}$

$F(a) < g(a)$

$F(b) > g(b)$

Take $[0,1]$

$F(0) < g(0)$

$F(1) > g(1)$

$$f(x) = \begin{cases} 0 & x = 0 \\ x & 0 < x < 1 \\ 1 & x = 1 \end{cases}$$

$$f(x) = \begin{cases} 0 & x = 0 \\ 1-x & 0 < x < 1 \\ 0 & x = 1 \end{cases}$$

$c \in (a,b)\ c \in (0,1) \quad F(c) = c, g(c) = 1 - c$

$\therefore f(c) + g(c) = 1 \neq 0$

$\therefore f(c) - g(c) = 2c - 1 \neq 0$

$\therefore f(c) \neq g(c)$

$\Rightarrow$ Option (4) is correct

3. **Explanation:**

$f(x) = \sin x$ is u.c on $[0, \infty]$

And also composition of uniform continuous function is uniformly continuous

$\therefore K(x) = \sin(\sin x)$ U.C on $[0, \infty]$

$\Rightarrow$ option (4) is correct

4. **Explanation:**

$f(x) = |x - c|^{\alpha}$ IS DIFFERENTIABLE IFF $\alpha > 1$

5. **Explanation:**

Function $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & when\ x \neq 0 \\ 0 & when\ x = 0 \end{cases}$

$f(0) = \lim_{x \to 0} F(x) \Rightarrow$ continuous

$\lim_{x \to 0} f(x) = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$

$\sin\left(\frac{1}{x}\right)$ is bounded function

$= \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

$f(0) = 0$

$f(0) = \lim_{x \to 0} f(x)$

$\therefore$ function $f(x)$ is continuous at $x = 0$

$\Rightarrow$ option (1) is correct

6. **Explanation:**

By Leibnitz rule of differentiation and integral

$1 + f(x) - f(0).0 = e^{x-1}$

$1 + f(x) = e^{x-1}$

$f(x) = e^{x-1} - 1, 1 \leq f(x) \leq 2$

i.e. $1 \leq e^{x-1} - 1 \leq 2$

i.e. $2 \leq e^{x-1} \leq 3$

i.e. $\log 2 \leq x - 1 \leq \log 3$

i.e. $1 + \log 3 \leq x \leq 1 + \log 3$

i.e. $\log e + \log 2 \leq x \leq \log e + \log 3$

i.e. $2e \leq x \leq 3e$

for $e^x - 1 = f(x)$

$2 \leq e^x \leq 3$

$\log 2 \leq x \leq \log 3$

[PRACTICE QUESTIONS]

1. **Explanation:**

Function $\sin x$ has derivative $\cos x$ which is bounded

$\Rightarrow f(x) = \sin x$ is uniformly continuous on $[0, \infty)$

$''y\ f(x) = e^{-x}$ on $[0, \infty)$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} e^{-x} = 1$

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} e^{-x} = 0$

Function $f(x) = e^{-x}$ is uniformly continuous at $[0, \infty$

$\Rightarrow$ option (2) is correct

2. **Explanation:**

$f(x) = x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)\right)$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} x\left(\sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x}\right)\right)$

$= \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) + x \cos\left(\frac{1}{x}\right).$

We know $\sin\left(\frac{1}{x}\right)$ is bounded.

As well as $\cos\left(\frac{1}{x}\right)$ is bounded.

$\therefore \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) + \lim_{x \to 0} x \cos\left(\frac{1}{x}\right)$

$= 0 + 0$

$\lim_{x \to 0} f(x) = 0$

$\Rightarrow$ option (3) is correct

3. **Explanation:**

$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}$

$\frac{1}{2} < \lim_{x \to 0} \frac{\sin x}{2x} < \frac{1}{2}$

$\frac{1}{2} < \lim_{x \to 0} \frac{\cos x}{2x} < \frac{1}{2}$

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$\frac{1}{2} < \frac{\cos(0)}{2} < \frac{1}{2} \Rightarrow \frac{1}{2} < \frac{1}{2} < \frac{1}{2}$

$\therefore \lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) = \frac{1}{2}$

4. Explanation:

$$f(x) = \begin{cases} x^2 & x < 2 \\ 3 & x = 2 \\ 2 & x > 2 \end{cases}$$

$\lim_{x\to 2^+} f(x) = f(2) = \lim_{x\to 2^-} f(x).$

$\lim_{x\to 2^+} 2 = 3 = \lim_{x\to 2^-} x^2$

$\lim_{x\to 2^+} 2 = 2\ f(0) = 3\ \lim_{x\to 2^-} x^2 = 4$

$\therefore \lim_{x\to 2^+} f(x) \neq 2 \neq 3 \neq 4$

$\therefore \lim_{x\to 2} f(x) = 3$ which required answer

$\Rightarrow$ option (4) is correct

5. Explanation:

$$f(x) = \frac{x(x^2-1)}{|x^2-1|} = \begin{cases} \frac{x(x^2-1)}{x^2-1} & x \geq 0 \\ \frac{x(x^2-1)}{-(x^2+1)} & x < 0 \end{cases}$$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(0).$

$\therefore \lim_{x\to 1^+} \frac{x(x^2-1)}{x^2-1} = 1\ \lim_{x\to 1^-} \frac{x(x^2-1)}{-(x^2-1)} = -1$

$\therefore \lim_{x\to -1^+} \frac{x(x^2-1)}{x^2-1} = -1\ \lim_{x\to -1^-} \frac{x(x^2-1)}{-(x^2-1)} = 1$

$\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$

y $\lim_{x\to -1^+} f(x) \neq \lim_{x\to -1^-} f(x)$

$\therefore f$ can not be continuously extended at $x = -1$

$\Rightarrow$ option (4) is correct.

6. Explanation:

$f(x) = \sin x$

For all $x, y \in \mathbb{R}$

$|\sin(x) - \sin(y)| = 2\left|\sin\left(\frac{x-y}{2}\right)\right|\left|\cos\left(\frac{x+y}{2}\right)\right|$

$\leq 2|12(x-y)|$

$= |x-y|$

$\therefore g(x) = \sin x$ is lipschitz on R

& Hence $\sin x$ uniformly continuous. Next $f(x) = x^2$

$|x^2 - y^2| \leq c|x-y|\ \forall\ x, y \in \mathbb{R}$

$\Rightarrow |x-y|.|x+y| \leq c|x-y|\ \forall\ x, y \in \mathbb{R}$

$|x+y| \leq c$ for all $x, y \in \mathbb{R}$

But there is no real no. C with property that $|x+y| \leq c\ \forall\ x, y \in \mathbb{R}$.

g is not lipschitz

g is not uniformly continuous.

$\Rightarrow$ option (2) is correct.

7. Explanation:

$g_n(x) = \frac{1}{(n(1+x^2))}\ n \geq 1$

$g_n(x) = \frac{1}{n(1+x^2)}$

$\lim_{n\to\infty} g_n(x) = \lim_{n\to\infty} \frac{1}{n^2+nx^2} = 0$

$\langle f_n(x)\rangle$ converges pointwise to 0

Now

$M_n = \sup_{x\in\mathbb{R}} |g_n(x) - g(x)|$

$= \sup_{x\in\mathbb{R}} \left|\frac{1}{n(1+x^2)} = 0\right|$

$= \left|\frac{1}{n(1+x^2)}\right| < \frac{1}{n}$

$\lim_{n\to\infty} M_n \to 0$ as $n - \infty$

$\Rightarrow E_1[g_n(x)]$ converge uniformly

$fn(x) = \frac{x+nx}{n}\ n \geq 1$

$\lim_{n\to\infty} \frac{x+nx}{n} = \lim_{n\to\infty} \frac{\left(\frac{x}{n}+x\right)}{n}$

$\langle fn(x)\rangle$ converges pointwise to x

Now

$M_n = \sup_{x\in\mathbb{R}} |fn(x) - f(x)|$

$= \sup_{x\in\mathbb{R}} \left|\frac{x+nx}{n} - x\right|$

$= \sup_{x\in\mathbb{R}} \left|\frac{x+nx-nx}{n}\right|$

$= \left|\frac{x}{n}\right|$

$\therefore \lim_{n\to\infty} M_n = \lim_{n\to\infty} \left|\frac{x}{n}\right| = 0$

$M_n \to 0$ as $n \to \infty$

$\Rightarrow \langle f_n(x)\rangle$ converges uniformly

$\Rightarrow$ option (2) is correct

8. Explanation:

The set of points where f is discontinuous is a countable set

9. Explanation:

$f : \mathbb{R} - \mathbb{R}$ is continuous

$f(x+y) = f(x) + f(y)\ \forall\ x, y \in \mathbb{R}.$

$f(0+0) = f(0) + f(0) = 0$

$\Rightarrow f(0) = 0$

$f(1) = C\ n > 0$ be an integer

To show : $f(x) = xf(1).$

$f(x-x) = 0 = f(x) + f(-x)$

$f(-x) = c(-x)$

$n \in \mathbb{Z} \Rightarrow f(x) = f(1)x$

$\Rightarrow$ option (3) is correct

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10. Explanation:

$f: \mathbb{R} \to \mathbb{R}$ be continuous & $f(a) = 0\ \forall\ q \in Q$ then for all $x \in \mathbb{R}$

f is continuous on $\mathbb{R}\ x_0 \in Q \subseteq \mathbb{R}$

$\forall \in > 0 \exists \delta > 0$ s.t.

$x \in \mathbb{R}\ |x - x_0| < \delta$ implies

$|f(x) - f(x_0)| < \delta$ implies

$|f(x) - f(x_0)| < \in$

If $x \notin \leq |a|$

But we must able to pick any $\varepsilon > 0$

Therefore, this is a contradiction Thus $f(x) \neq a\ \forall x \notin Q \iff f(x) = 0\ \forall\ x \notin Q$

Hence $f(x) = 0\ \forall x \in \mathbb{R}$

$\Rightarrow$ option (1) is correct

11. Explanation:

The function f defined $f(x, y) = |x| + |y|$

$\lim\limits_{\substack{x\to 0^+ \\ y\to 0^+}} f(x, y) = \lim\limits_{\substack{x\to 0^+ \\ y\to 0^+}} (|x| + |y|) = 0 + 0 = 0$

$\lim\limits_{\substack{x\to 0^- \\ y\to 0^-}} f(x, y) = \lim\limits_{\substack{x\to 0^- \\ y\to 0^-}} (-x - y) = 0.0 = 0$

$\lim\limits_{\substack{x\to 0^- \\ y\to 0^-}} f(x, y) = f(0,0) = |0| + |0| = \lim\limits_{\substack{x\to 0^- \\ y\to 0^-}} f(x, y)$

$\Rightarrow f(x, y)$ is continuous at origin

$\lim\limits_{\substack{x\to 0^+ \\ y\to 0^+}} f'(x, y) = 1 + 1 = 2$

$\lim\limits_{\substack{x\to 0^+ \\ y\to 0^+}} f'(x, y) = -1 - 1 = -2$

$\therefore f(x, y)$

Is not differentiable at origin

$\Rightarrow$ option (3) is correct

12. Explanation:

The $\tan x$ is continuous at every point of the domain except $x = 0$

$\Rightarrow$ option (4) is correct

13. Explanation: $f(x) = \frac{1}{1+x^n} \forall x \in [0,1]$

$$= \begin{cases} 1 & ; x = 0 \\ \frac{1}{1+x^n} & ; x \in (0,1) \\ \frac{1}{2} & ; x = 1 \end{cases}$$

(as for $0 < x < 1, \lim\limits_{n\to\infty} x^n = 0$)

Note that $f(x)$ is not continuous at $x = 1$, and is continuous for all $x \in [0, a]$, *where* $a < 1$.

Since, $f(x)$ is continuous for all, n hence $f_n(x)$ converges uniformly to $f(x)$ on $[0, a]$ where $a < 1$.

Hence option (1) is correct.

14. Explanation: Here, $f_n(x) = \frac{\sin x}{x} + \frac{\cos x}{\sqrt{n}} \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore f(x) = \lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty} \left(\frac{\sin x}{x} + \frac{\cos x}{\sqrt{n}}\right)$

$= \frac{\sin x}{x} + \lim\limits_{n\to\infty} \frac{\cos x}{\sqrt{n}}$

Since, $0 \leq \cos x \leq 1 \forall\ x \in \left(0, \frac{\pi}{2}\right]$ and $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} = 0$,

therefore,

$\lim\limits_{n\to\infty} \frac{\cos x}{\sqrt{n}} = 0$

$\Rightarrow f(x) = \frac{\sin x}{x} \forall x \in \left(0, \frac{\pi}{2}\right]$

Now, $||f_n(x) - f(x)|| = \left|\left|\frac{\cos x}{\sqrt{n}}\right|\right| \leq \frac{1}{\sqrt{n}} \forall\ n \in \mathbb{N}$

Since, $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ then $f_n(x)$ converges to $f(x)$ uniformly

Hence option (2) is correct

15. Explanation: For simplicity let us assume that $f(x) = k\ \forall\ x \in [a, b]$, then

$\lim\limits_{n\to\infty} \int_a^b f(x) \sin nx dx = \lim\limits_{n\to\infty} \int_a^b k \sin nx dx$

$= \lim\limits_{n\to\infty} \frac{-k}{n} [\cos nb - \cos na]$

Note that, $|\cos nb - \cos na|$ is bounded

So, $\lim\limits_{n\to\infty} \frac{-k}{n} [\cos nb - \cos na] = 0$

Similarly, if we replace $\sin nx$ by $\cos nx$ we have

$\lim\limits_{n\to\infty} \int_a^b f(x) \cos nx dx = \lim\limits_{n\to\infty} \int_a^b k \cos nx dx = 0$

Hence option (C) and (D) are correct.

16. Explanation:

For option: (A) $f_n(x) = (1 - |x|)^n : x \in (-1,1)$

$$= \begin{cases} (1 - |x|)'' & : x \in (-1,1)\{0\} \\ 1 & : x = 0 \end{cases}$$

$$\therefore f(x) = \lim_{n\to\infty} f(x) = \begin{cases} 0\ if\ x \in (-1,1)\backslash\{0\} \\ 1\ if\ x = 0 \end{cases}$$

Clearly, $f_n(x)$ is continuous function for all , But $f(x)$ is not continuous

Hence $f_n(x)$ do not converges uniformly.

For option: (B) $f_n(x) = \frac{1}{n} \sin nx, x \in \mathbb{R}$

$\therefore f(x) = \lim\limits_{n\to\infty} \frac{\sin nx}{n}$

Clearly, $f_n(x)$ is continuous function for all n, Also $f(x)$ is continuous. Hence, $f_n(x)$ converges uniformly

For option:(C) $f(x) = x^n \forall x \in [0,1]$

$$= \begin{cases} 0 : x = 0 \\ x^n : 0 < x < 1 \\ 1 : x = 1 \end{cases}$$

$$\therefore f(x) = \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \begin{cases} x^n : 0 \leq x < 1 \\ 1 : x = 1 \end{cases}$$

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Clearly, $f_n(x)$ is continuous for all x, but $f(x)$ is not hence $f_n(x)$ cannot converge uniformly.

For option: (D)

$f_n(x) = \frac{1}{1+x^n}; x \in [0, \infty)$

$$f(x) = \begin{cases} \frac{1}{1+x^n}; 0 \le x < 1 \\ \frac{1}{2}; x = 1 \\ \frac{1}{1+x^n}; x > 1 \end{cases}$$

$$\therefore \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \begin{cases} \frac{1}{1+x^n}; 0 \le x < 1 \\ \frac{1}{2}; x = 1 \\ \frac{1}{1+x^n}; x > 1 \end{cases}$$

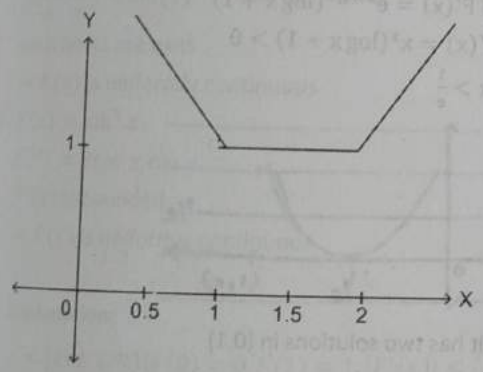
$$\begin{cases} 1; 0 \le x < 1 \\ \frac{1}{2}; x = 1 \\ 1; x > 1 \end{cases}$$

Clearly, $f_n(x)$ is continuous for all x, but $f(x)$ is not continuous. Hence $\langle f_n(x) \rangle$ cannot converge to $f(x)$ uniformly.

Hence option (2) is correct.

17. Explanation

$f: \mathbb{R} \to \mathbb{R}, f(x) = |x-1| + |x+2|$



As the graph has sharp edge at 1 and 2.

$\therefore$ at $x = 1,2$ the graph is not differentiable

And it is continuous

$\Rightarrow$ option (2) is correct

18. Explanation:

$\lim_{x\to 0}(\cos x)^{(1/\sin^2 x)}$

$= e^{\lim_{x\to 0}\frac{1}{\sin^2 x}[\ln(\cos x)]}$

$= e^{\lim_{x\to 0}\frac{\ln(\cos x)}{\sin^2 x}}$

$= e^{\lim_{x\to 0}\frac{\frac{1}{\cos x}(-\sin x)}{2\sin x\cos x}}$

$= e^{\lim_{x\to 0}\frac{-1}{2\cos^2 x}}$

$= e^{\lim_{x\to 0}\frac{-1}{1+\cos 2x}}$

$= e^{-1/2}$

$\Rightarrow$ option (3) is correct

19. Explanation:

$$\lim_{x\to a}|x^2-1| = \lim_{x\to a} 0$$

$$a^2 - 1 = 0$$

$$a = \pm 1$$

The point of continuity of the function $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} |x^2-1| & if\ x\ is\ irrational \\ 0, & If\ is\ rational \end{cases} are$$

(2) $x = -1, x = 1$

20. Explanation:

$f, g: (0,1) \to \mathbb{R}$

$f(x) = x \sin\left(1/x^2\right)$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x \sin\left(1/x^2\right) = 0$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} x \sin\left(1/x^2\right) = \sin(1)$

$\Rightarrow f(x)$ is U.C

And

$g(x) = x^2$

$\lim_{x\to 0} g(x) = \lim_{x\to 0} x^2 = 0$ and

$\lim_{x\to 1} g(x) = 1$, both limit exists

$\Rightarrow$ U.C

$\Rightarrow$ option (1) is correct

21. Explanation Let $f(x) = c$, c-const2ant function and $\lim_{n\to 0} f(x+n) - f(x-n) = 0$ and $f'(x) = 0$

Which is continuous every where

$\Rightarrow$ f is differentiable and f' is continuous

$\Rightarrow$ option (4) is correct

22. Explanation:

$\lim_{x\to 0}\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$

$= \lim_{x\to 0}\frac{e^{1/x}\left(1 - e^{-2/x}\right)}{e^{1/x}\left(1 + e^{-2/x}\right)}$

$= \lim_{x\to 0}\frac{1 - e^{-2/x}}{1 + e^{-2/x}}$

$= 1$

$\Rightarrow$ option (2) is correct

23. Explanation:

$\lim_{x\to 0^+}\left(1 + \frac{1}{x}\right)^{2x} = e^{\lim_{x\to 0^+} 2x\left(1+\frac{1}{x}-1\right)}$

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$= e^2$

c) $F(x, y) = |x|$

$\Rightarrow F(x, y)$ is continuous at $(x, y) = 0$

$F_x = \lim_{x\to 0^+} \frac{|x|}{x} = 1$

And $\lim_{x\to 0^-} \frac{|x|}{x} = -1$

$\Rightarrow F_x$ does not exists

24. Explanation:

$$\lim_{x\to 1} \frac{(1+x)^{1/x}}{x-1}$$

By L-Hospital rule

$$= \lim_{x\to 1} \frac{\frac{d}{dx}\left[(1+x)^{1/x-2}\right]}{\frac{d}{dx}(x-1)}$$

$$= \lim_{x\to 1} \frac{d}{dx}(1+x)^{1/x}$$

Let $y = (1+x)^{1/x}$

$\log y = \log(1+x)^{1/x}$

$\log y = \frac{1}{x}\log(1+x)$

$\frac{1}{y}y' = \frac{\frac{x}{1+x} - \log(1+x).1}{x^2}$

$y' = y\left[\frac{x-(1+x)\log(1+x)}{x^2(1+x)}\right]$

$y' = (1+x)^{1/x}\left[\frac{x-(1+x)\log(1+x)}{x^2(1+x)}\right]$

$\therefore \lim_{x\to 1} y' = \lim_{x\to 1}(1+x)^{1/x}\left[\frac{x-(1+x)\log(1+x)}{x^2(1+x)}\right]$

$= 2\left[\frac{1-2\log 2}{2}\right]$

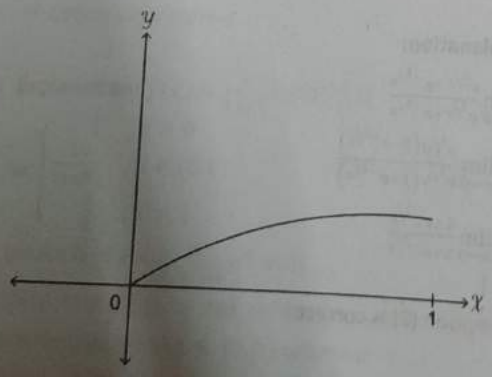
$= 1 - 2\log 2$

25. Explanation:

1) Given $F(x) = \sqrt{x}$

$$F'(x) = \frac{1}{2\sqrt{x}}$$

$\therefore$ F is diff.



2) as $\lim_{\alpha\to 0^+} F'(x) \to \infty$

F' is not bdd

3) Now, By continuous extension then, we have

$F : (a, b) \to \mathbb{R}$ S.t F is continuous then F is V.C iff f(a) and f(b) is defined

$\Rightarrow$ F is V.C

4) $F' = \frac{1}{2\sqrt{x}}$, $\forall x \in (0,1)$ is not V.C

On (0, 1) as it cannot be defined

26. Explanation:

Given $x^x = \left(\frac{1}{2}\right)^{1/2}$

Let $F(x) = x^x = e^{x\log x}$

Now at (0,1) we have

$F(0) = 0^0$

$\therefore \lim_{x\to 0} x^x = \lim_{x\to 0} e^{x\log x}$

$= e^{\lim_{x\to 0}\frac{\log x}{1/x}}$

$= e^{\lim_{x\to 0}\frac{\left(\frac{1}{x}\right)}{-1/x}}$

$= e^{\lim_{x\to 0}(-x)}$

$= e^{-0}$

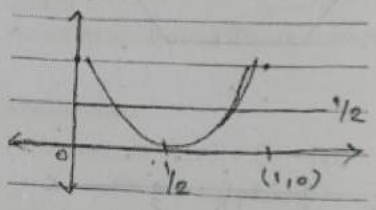
$= 1$

And $F(1) = 1^1 = 1$

And $F'(x) = e^{x\log x}(\log x + 1)$

$\therefore F'(x) = x^x(\log x + 1) > 0$

$\Rightarrow x > \frac{1}{e}$



$\Rightarrow$ It has two solutions in (0,1)

27. Explanation:

(a) $\in > 0\ \exists \delta > 0$ such that

$|x - y| < \delta \Rightarrow |f(x) - f(y)| < \in$

Consider $\delta = \in^2$

$$|x - y| < \delta \Rightarrow |f(x) - f(y)| \leq |x - y|^{\frac{1}{2}} = \delta^{\frac{1}{2}} \in$$

28. Explanation: If $f(x) = \begin{cases} \phi_1(x) & x\epsilon Q \\ \phi_2(x) & x\epsilon Q^c \end{cases}$

Then $f(x)$ is continuous at α such that

$\lim_{x\to\alpha} \phi_1(x) = \lim_{x\to\alpha} \phi_2(x)$

(1) $f(x) = \begin{cases} 1 & x\epsilon Q \\ 0 & x\epsilon Q^c \end{cases}$

$\lim_{x\to\alpha} 1 = \lim_{x\to\alpha} 0$

Is not possible for any $x \epsilon lR$

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$\Rightarrow f(x)$ is not continuous

(2) $f(x) = \begin{cases} x & x \epsilon Q \\ 0 & x \epsilon Q^c \end{cases}$

$\lim_{x \to a} x = \lim_{x \to a} 0$

$a = 0$

$\Rightarrow f(x)$ is continuous only at $x = 0$

(3) $f(x) = \begin{cases} sin\pi x & x \epsilon Q \\ 0 & x \epsilon Q^c \end{cases}$

$\lim_{x \to a} \sin \pi x = \lim_{x \to a} 0$

$\sin \pi a = 0$

$\Rightarrow a = n \quad n \epsilon \mathbb{Z}$

$\Rightarrow F(x)$ continuous at contable point

29. Explanation:

(1) $F(x) = \frac{1}{x}\ x \epsilon (0,1)$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} + \frac{1}{x} = DNE$

$\Rightarrow f(x)$ is not U.C

(2) $F(x) = \frac{sinx}{x} \qquad x \epsilon (0,1)$

$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} cosx = 1$

$\lim_{x \to 0} \frac{\sin x}{x} = \sin(1)$

Both limits are exits

$\Rightarrow F(x)$ is uniformly continuous

(3) $F(x) = \sin^2 x$

$F^{\prime(x)} = 2\sin\ x \cos x$

$F'(x)$ is bounded

$\Rightarrow F(x)$ is uniformly continuous

30. Explanation:

$A = \left\{F \epsilon C'(IR) | F(0) = 0, F(1) = 1, |F'(x)| \leq \frac{1}{2} \forall x \in |R\right\}$

$\exists$ any function such that

$F(0) = 0, f(1) = 1, |F'(x)| \leq \frac{1}{2}$

So A is empty set

(c)

Here,

$a_n = \frac{(-1)^n n}{n+2}$

$|a_n| = \frac{n}{n+2}$

$\lim_{n \to \infty} |a_n| = 1$

$\Rightarrow \sum |an|$ is divergent

$\Rightarrow \sum |an|$ is not converges absolutely

31. Explanation:

(1) $F(x) = \sqrt{x} \qquad 1 \leq x \leq 2.$

$\lim_{x \to 1} f(x) = 1 \qquad \& \qquad \lim_{x \to 2} f(x) = \sqrt{2}$

$\Rightarrow$both limit exits Finitely.

$\Rightarrow F(x)$ is uniformly continuous by continuous extension theorem

(2) $f(x) = x^3 \qquad x \in IR$

$\lim_{x \to \infty} f(x) = DNE$

$\lim_{x \to \infty} f(x) = DNE$

$\Rightarrow f(x)$ is not uniformly continuous

(3) $F(x) = \sin^2 x.$

$F'(x) = 2\sin\ x \cos x\ .$

$F'(x)$ is bounded

$\Rightarrow F(x)$ is uniformly continuous.

32. Explanation:

(1) $F(x) = \sin^2 x$

$F'(x) = 2 \sin x \cos x \qquad x \in IR$

$F'(x)$ is bounded

$\Rightarrow F(x)$ is uniformly continuous

(2) $F(x) = x \sin \frac{1}{x}$

$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$

$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$

$\Rightarrow F(x)$ is uniformly continuous

(3) $F(x) = x^2 \qquad x \in IR$

$\lim_{x \to \infty} F(x) = \lim_{x \to \infty} x^2 = DNE$

$\lim_{x \to \infty} F(x, y) = \lim_{x \to \infty} x^2 = DNE$

$\Rightarrow$ Vy continuous extension theorem

$F(x)$ is $U.C$

33. Explanation:

(2) $F(x) = \lim_{n \to \infty} \frac{x^2 e^{nx} + x}{e^{nx} + 1} \qquad x \in IR$

$= \lim_{n \to \infty} \frac{nx^2 e^{nx}}{ne^{nx}}$

$F(x) = x^2$

$\Rightarrow F(x)$ is continuous

(3) $F(x) = \lim_{n \to \infty} \frac{1}{n} \log(e^n + x^n) \qquad x \geq 0$

$= \lim_{n \to \infty} \left(\frac{e^n + x^n \log x}{e^n + x^n}\right)$

$= \lim_{n \to \infty} \frac{e^n + x^n \log x \log x}{e^n + x^n \log x} = 1$

F(x) is continuous

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34. Explanation:

$f(x) = \sin\frac{1}{x}$

$f(x) = x\sin\frac{1}{\sqrt{x}}$

35. Explanation:

(1) $f(x) = [x^2]\sin\pi x \qquad x > 0$

Here $\sin x$ is continuous everywhere

& $[x^2]$ is discontinuous At Integers

$i.e\ x^2 = n$

$x = \sqrt{n}$

$\Rightarrow f(x)$ is discontinuous at $x = \sqrt{n}$

36. Explanation:

(1) $f(x) = \cos x\cos\left(\frac{\pi}{x}\right)\ x \in (0\ 1)$

Let $a_n = \frac{1}{n} \to 0$ as $n \to \infty$

$\lim_{n\to\infty} f\left(\frac{1}{n}\right) = \cos\left(\frac{1}{n}\right)\cos(n\pi) = (-1)^{n+1} \neq 0$

$\Rightarrow f\left(\frac{1}{n}\right) \nrightarrow 0$ as $n \to \infty$

$f(x)$ is not uniformly continuous

(2) $f(x) = \sin x\cos\frac{\pi}{x}$

$\sin x\cos\left(\frac{\pi}{x}\right)$ is continuous on (0,1]

& $\lim_{x\to 0^+} \sin x\cos\left(\frac{\pi}{x}\right) = 0$

$\Rightarrow f(x)$ is uniformly continuous

(3) $f(x)\sin^2 x \qquad x \in [0,\infty)$

$f(x) = 2\sin x\cos x$

Since $f(x)$ is bounded

$\Rightarrow f(x)$ is uniformly continuous

37. Explanation:

(1) $F: [-1,1] \to IR$

$f(x) = \begin{cases} 1, & x \in QC \\ 0, & x \in Q \end{cases}$

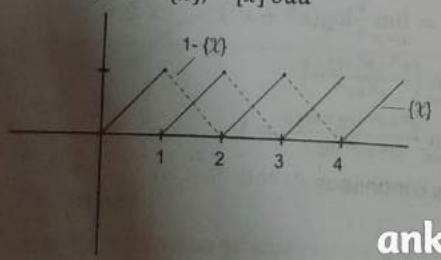
$\lim_{x\to\alpha} x = \lim_{x\to\alpha} 0$

$\alpha = 0$

$\Rightarrow f(x)$ is continuous at $\alpha = 0$

Set of discontinuities $[-1,1]\{0\}$

(3) $f(x) = \begin{cases} \{x\}, & if \quad [x]\ even \\ 1-\{x\}, & [x]\ odd \end{cases}$



$f(x)$ is continuous

Set of discontinuities is empty

38. Explanation: $F: \mathbb{R} \to \mathbb{R}$

$F(x) = \begin{cases} 0 & x \in Q \\ \sin|x| & x \in Q^c \end{cases}$

$\lim_{x\to\alpha} 0 = \lim_{x\to\alpha} \sin|x|$

$\sin|\alpha| = 0$

$|\alpha| = \pm n\pi$

$\alpha = n\pi \qquad n \in \mathbb{Z}$

39. Explanation:

$f(x) = \begin{cases} a_n + \sin\pi x & if\ xe(2n, 2n+1) \\ b_n + \cos\pi x & if\ xe(2n-1,2n) \end{cases}$

$\lim_{x\to 2n^+} f(x) = \lim_{x\to 2n^-} (a_n + \sin\pi x)$

$\lim_{x\to 2n^+} f(x) = a_n$

$\lim_{x\to 2n^-} f(x) = \lim_{x\to 2n^-} (bn + \cos\pi x)$

$= bn + 1$

For continuity

$\lim_{x\to 2n} f(x) = \lim_{x\to 2n^-} f(x)$

$a_n = b_n + 1$

40. Explanation:

(1) $f(x) = x\sin\frac{1}{x}$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x\sin\frac{1}{x} = 0$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} x\sin 1 = \sin(1)$

Exists finitely

By continuous extension theorem $f(x)$ is v.c

$f(x) = e^x\cos\left(\frac{1}{x}\right)\ x \in (0,1)$

$x_n = \frac{1}{2\pi n}, y_n = \frac{2}{\pi+\pi^n}$

$|x_n - y_n| = \frac{1}{2n\pi(n+4)} \to 0$ as

$\left(f(x_n) - f(y_n)\right) = \left|e^{\frac{1}{2\pi n}}\cos(2\pi n) - e^{\frac{2}{\pi+4\pi n}}\cos\left(\frac{\pi+4\pi}{2}\right)\right|$

$= \left|e^{\frac{1}{2\pi n}}\right| \to 1$ as $n \to \infty$

$f(x)$ is not uniformly continuous.

(3) $f(x) = \sin(\sin x)\ x \in (0,\infty)$

$f'(x) = \cos(\sin x)\cos x$

$f'(x)$ is bounded then

$f(x)$ is U./c

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41. Explanation:

Let r be radius and h be the height of the cylinder having its total surface area A since cylindrical container is closed at the top (circular) then its surface area

= (area of lateral surface) $+2\left[\frac{area\ of\ circular}{bottom}\right]$

$A = 2\pi rh + 2\pi r^2$

$h = \frac{A}{2\pi r} - r$

$V = \pi r^2 h$

$= \pi r^2\left(\frac{A}{2\pi r} - r\right)$

$= \frac{Ar}{2} - \pi r^2$

Diff. w.r+ r

$\frac{dv}{dr} = \frac{A}{2} - 3\pi r^2$

$\frac{d^2V}{dr^2} = -6\pi r < 0 \quad \forall r > 0$

Hence the volume is max

$\frac{dv}{dx} = 0$

$\frac{A}{2} - 3\pi r^2 = 0$

$r = \sqrt{\frac{A}{6\pi}}$

$h = \frac{A}{2\pi r} - r$

$= \frac{A}{2\pi\sqrt{\frac{A}{6\pi}}} - \sqrt{\frac{A}{6\pi}}$

$\sqrt{\frac{2A}{3\pi}}$

Ratio of radius and height

$\frac{h}{r} = \frac{\sqrt{\frac{2A}{3\pi}}}{\sqrt{\frac{A}{6\pi}}} = 2$

$\frac{h}{r} = 2$

$h = 2r$

42. Explanation:

$J_n = \int_0^{\pi/2} \cos^n x\, dx$

J_n n is even then

$J_n = \frac{(n-1)(n-3)--(1)}{n(n-2)--(2)} \cdot \frac{\pi}{2}$

J_n n is odd then

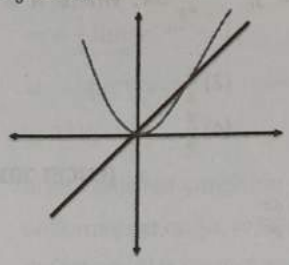
$\frac{(n-1)(n-3)--(1)}{n(n-2)--(2)}$

How 7 is odd then J_n is rational

→ only c is true

43. Explanation:

$\int_0^5 \max\{x^2, 6x-8\}dx$



$\Rightarrow J_n(0,2) \quad F(x) = x^2$

$J_n(2,4) \quad F(x) = 6x - 8$

$J_n(4,5) \quad F(x) = x^2$

$\int_0^5 \max\{x^2, 6x-8)\}dx$

$\int_0^2 x^2dx + \int_2^4 (6x-8)dx + \int_4^5 x^2dx$

$= \left[\frac{x^3}{3}\right]^2 + [3x^2 - 8x]_2^4 + \left[\frac{x^3}{0}\right]_4^5$

$= \frac{8}{3} + [48 - 32 - 12 + 16] + \left[\frac{125-64}{3}\right]$

$= \frac{8}{3} + [20] + \left(\frac{61}{3}\right)$

$= \frac{69}{3} + 20 = 43$

44. Explanation: $F(x) = e^{\frac{1}{x}}$

(1) (0 1)

$\lim_{x\to 0} e^{\frac{1}{x}} = DNE$

$\Rightarrow e^{\frac{1}{x}}$ is not uniformly continuous

(2) $(1, \infty)$

$\lim_{x\to 1} e^{\frac{1}{x}} = e$

$\lim_{x\to\infty} e^{\frac{1}{x}} = 1$

By continuous extension theorem $F(x) = e^{\frac{1}{x}}$ is u.c on$(1, \infty)$

(3) (1,2)

$\lim_{x\to 1} e^{\frac{1}{x}} = e$

$\lim_{x\to 2} e^{\frac{1}{x}} = e^{\frac{1}{2}}$

By continuous extension both limit are bounded

$\Rightarrow e^{1/x}$ is u.c on (1,2)

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[CU-CET]

[CUCET 2016]

1. The value of the integral $\int_0^{\pi/2} \frac{x}{x+x} dx$, where n is a natural number is
   (1) π (2) $\frac{\pi}{2}$
   (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

[CUCET 2016]

2. The improper integral $\int_1^{\infty} \frac{dx}{x^n}$
   (1) converges if $n < 1$
   (2) converges in $n = 1$
   (3) converges if $n > 1$
   (4) converges for all values of n

[CUCET 2016]

3. The improper integral $\int_0^{\infty} sinx\, e^{-xy} dx$, where $y > 0$ satisfies which one of the following?
   (1) does not converge for some $y > 0$
   (2) converges for all $y > 0$ but not uniformly on $[a, \infty)$ for any $a > 0$
   (3) converges uniformly on $(0, \infty)$
   (4) converges uniformly on $[a, \infty)$, where $a > 0$

Answer Key		
1	2	3
4	3	4

[PRACTICE QUESTIONS]

1. Suppose $f: [0,1] \to \mathbb{R}$ is bounded. Then
   (1) f is Riemann integrable on [0, 1]
   (2) f is continuous on (0,1) except for finitely many points implies f is Riemann integrable on [0, 1]
   (3) f is Riemann integrable on [0,1] implies is continuous on [0, 1]
   (4) f is Riemann integrable on [0, 1] implies f is monotone function.

2. Let $f: [0,1] \to \mathbb{R}$ be defined by
$$f(x) = \begin{cases} \frac{1}{2} & if\ x = 1/4 \\ \frac{1}{4} & if\ x = 1/2 \\ 0 & if\ x \in [0,1]\{1/4, 1/2\} \end{cases}$$
   Then
   (1) f is Riemann integrable and $\int_0^1 f(x)dx = 3/4$
   (2) f is Riemann integrable and $\int_0^1 f(x)dx = 1/4$
   (3) f is Riemann integrable and $\int_0^1 f(x)dx = 0$
   (4) f Riis not Riemann integrable.

3. Which one of the following integrals is convergent?
   (1) $\int_1^{\infty} \frac{1}{x^2} dx$ (2) $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$
   (3) $\int_0^1 \frac{1}{x^2} dx$ (4) $\int_0^{\infty} \frac{1}{\sqrt{x}} dx$

4. The value of the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$ is
   (1) 0 (2) $\sqrt{2\pi}$
   (3) $\sqrt{\pi}$ (4) $\sqrt{\pi/2.}$]

5. Consider the function $f(x) = x^2$ on, [0,1] and the partition of [0,1] given be
   $P = \left\{0 < \frac{1}{n} < \frac{2}{n} < .. < \frac{n-1}{n} < 1\right\}$
   Then the upper and the lower Riemann sums of f are
   (1) $U(f,P) = \left(1+\frac{1}{n}\right)\left(2-\frac{1}{n}\right)/6$ and $L(f,P) = \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)/6$
   (2) $U(f,P) = \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)/6$ and $L(f,P) = \left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)/6$
   (3) $U(f,P) = \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)/6$ and $L(f,P) = \left(1-\frac{1}{n}\right)\left(2+\frac{1}{n}\right)/6$
   (4) $U(f,P) = \left(1-\frac{1}{n}\right)\left(2+\frac{1}{n}\right)/6$ and $L(f,P) = \left(1+\frac{1}{n}\right)\left(2-\frac{1}{n}\right)/6$

6. The integral $\int_0^{\infty} \sin x dx$
   (1) exists and equals 0. (2) exists and equal1.
   (3) exists and equals -1. (4) does not exist

7. The improper integral $\int_{-\infty}^{0} 2^x dx$ is
   (1) convergent and converges to 2
   (2) divergent
   (3) convergent and converges to $\frac{1}{\ln 2}$
   (4) convergent and converges to $\ln 2$

8. Let $f(x) = x^2, x \in [0,1]$. For each $n \in \mathbb{N}$, let P_n be the partition of [0,1] given by a

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$P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \ldots \frac{n}{n}\right\}$ .If $\alpha_n = U(f, P_n)$ (upper sum) and $\beta_n = L(f, P_n)$. (lower sum) then

(1) $\alpha_n = (n+2)(2n-1)/(6n^2)$

(2) $\beta_n = (n-2)(n+1)/(6n^2)$

(3) $\beta_n = (n-1)(2n-1)/(6n^2)$

(4) $\lim_{n\to\infty} \alpha_n \neq \lim_{n\to\infty} \beta_n$

9. Let $I = \int_0^{\pi/2} \log \sin x dx$. Then

(1) I diverges $x = 0$

(2) I converges and is equal to $-\pi \log 2$

(3) I converges and is equal to $-\frac{\pi}{2} \log 2$

(4) I diverges at $x = \frac{x}{4}$

Answer Key								
1	2	3	4	5	6	7	8	9
2	3	1	3	2	4	3	3	3

EXPLANATIONS

[CU-CET]

1. **Explanation:**

$\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = I$

$I = \int_0^{\pi/2} \frac{\cos^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)} dx$ (1)

$\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$= \int_0^{\pi/2} \frac{\sin^n(x)}{\cos^n x + \sin^n x} dx$ (2)

Adding (1) & (2) we get

$2I = \int_0^{\pi/2} \left(\frac{\cos^n x}{\cos^n x + \sin^n x} + \frac{\sin^n x}{\cos^n x + \sin^n x}\right) dx$

$2I = \int_0^{\pi/2} \left(\frac{\cos^n x + \sin^n x}{\cos^n x + \sin^n x}\right) dx$

$2I = \int_0^{\pi/2} dx$

$2I = \pi/2$

$I = \pi/4$

2. **Explanation:**

$\int_1^{\infty} \frac{dx}{x^n} = \begin{cases} diverges\ if\ p \leq 1 \\ converges\ if\ p > 1 \end{cases}$

⇒ converges above integer if $p > 1$

⇒ option (3) is correct

3. **Explanation:**

$\int_0^{\infty} e^{-xy} \sin x\, dx \qquad y > 0$

$|e^{-xy} \sin x| = |e^{-xy}||\sin x| \leq |e^{-xy}|$

$|e^{-xy} \sin x| \leq e^{-xy}$

$e^{-xy} \leq e^{-x\lambda}\ \lambda \leq y$

$m(x) = e^{-x\lambda}$

$\int_0^{\infty} m(x)\, dx = \lim_{\mu\to\infty} \int_0^{\mu} e^{-x\lambda}\, dx$

$= \lim_{\mu\to\infty} \left|\frac{e^{x\lambda}}{-\lambda}\right|_0^{\mu}$

$= -\frac{1}{\lambda} \lim_{\mu\to\infty} \left[e^{-\mu\lambda} - 1\right]$

$= -\frac{1}{\lambda}\left[e^{-\infty} - 1\right]$

$= \frac{-1}{\lambda}[0 - 1] = \frac{1}{\lambda}$

$M(x)$ satisfied condition of M-test $\int_0^{\infty} e^{-xy} \sin x\, dx$ is uniformly cgt on $[a, \infty)$ where $a > 0$

⇒ Option (4) is correct answer

[PRACTICE QUESTIONS]

1. **Explanation:** Given that, $f: [0,1] \to \mathbb{R}$ is bounded.

i.e., $f(x) \in [m, M] \forall x \in [0,1]$ and $m, M \in \mathbb{R}$

option (A) is not necessarily true as

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q} \cap [0,1] \\ 0, & x \in \mathbb{Q} \cap [0,1] \end{cases}$$

is discontinuous at every point of [0, 1], so can't be Riemann integrable.

And, option (B) is true as piecewise continuous functions are Riemann integrables.

Also, option (C) is not necessarily true, as it can be piecewise continuous

For example: $f(x) = \begin{cases} 1; x \in \left[\frac{1}{2}, 1\right] \\ 1; x \in \left[0, \frac{1}{2}\right) \end{cases}$ is Riemann integrable but not continuous

Also, option (D) is not true, as

$f(x) = \begin{cases} \frac{1}{2}; x \in \left[0, \frac{1}{3}\right] \\ 0; x \in \left(\frac{1}{3}, \frac{2}{3}\right) \\ 1: x \in \left[\frac{2}{3}, 1\right] \end{cases}$ is Riemann integrable but not monotone

Hence option (2) is correct.

2. **Explanation:** $f: [0,1] \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} \frac{1}{2}\ if\ x = \frac{1}{4} \\ \frac{1}{4}\ if\ x = \frac{1}{2} \\ 0\ if\ x \in [0,1]\left\{\frac{1}{4}, \frac{1}{2}\right\} \end{cases}$$

Clearly $f((x)$ is piecewise continuous function on [0, 1], hence Riemann integrable

And $\int_0^1 f(x)dx = \int_0^{1/4} f(x)dx + \int_{1/4}^{1/4} f(x)dx + \int_{1/2}^1 f(x)dx$

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[Note that integral do not depends on a point]

Since, $f(x) = 0$ for each of the case, so $\int_0^1 f(x)dx = 0 + 0 + 0 = 0$

Hence option (3) is correct.

3. **Explanation:** For (A), we have

$\int_1^\infty \frac{1}{x^2} dx = \left[\frac{x^{-2+1}}{-2+1}\right]_1^\infty = \lim_{t\to\infty}\left(-\frac{1}{t}+1\right) = 0+1 = 1$

Therefore integral converges

Hence, option (1) is correct

4. **Explanation:**

$I = \int_{-\infty}^{\infty} e^{-x^2} dx = 2\int_0^\infty e^{-x^2} dx$

Let, $x^2 = t$, then we have, $2x\,dx = dt$

$i.e.\ dx = \frac{dt}{2x} = \frac{1}{2}\frac{dt}{\sqrt{t}}$

$\therefore\ I = 2\int_0^\infty \frac{1}{2\sqrt{t}} e^{-1} dt$

$= \int_0^\infty t^{\frac{1}{2}-1} e^{-t} dt = \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

Hence option (3) is correct

5. **Explanation:**

$f(x) = x^2$ on $[0,1]$ and $P = \left\{0 < \frac{1}{n} < \frac{2}{n} < \cdots < \frac{n-1}{n} < 1\right\}$

Now, consider any subinterval $[t_{k-1}, t_k] = \left[\frac{k-1}{n}, \frac{k}{n}\right]$

We have $M[f,(t_{k-1}t_k)] = \max_{x\in[t_{k-1}t_k]} f(x) = t_k^2 = \frac{k^2}{n^2}$

And, $m[f,(t_{k-1},t_k)] = \max_{x\in[t_{k-1}t_k]} f(x) = t_{k-1}^2 = \frac{(k-1)^2}{n^2}$

$\therefore U(f,P) = \sum_{k=1}^n M(f,[t_{k-1},t_k])(t_k - t_{k-1})$

$\sum_{k=1}^n \frac{k^2}{n^2} \times \frac{1}{n} = \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$= \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$

Also, $L(f,P) = \sum_{k=1}^n m(f,[t_{k-1},t_k])(t_k - t_{k-1})$

$\Rightarrow \sum_{k=1}^n \frac{(k-1)^2}{n^2}\cdot\frac{1}{n} = \frac{1}{n^3}\cdot\frac{(n-1)(n)(2n-1)}{6}$

$= \frac{\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}$

Hence option (2) is correct

6. **Explanation:**

$I = \int_0^\infty \sin x dx = \lim_{t\to\infty}\int_0^t \sin x dx$

$\lim_{t\to\infty}[-\cos x]_0^t = \lim_{t\to\infty}[-\cos t + 1]$

Since, $\lim_{t\to\infty} \cos t$ do not exist, hence improper integral diverges.

Hence option (4) is correct

7. **Explanation:**

$I = \int_{-\infty}^0 2^x dx = \lim_{y\to\infty}\left[\frac{2^x}{\ln 2}\right]_y^0 = \lim_{y\to\infty}\left[\frac{1}{\ln 2} - \frac{2^y}{\ln 2}\right]$

Since, $\lim_{y\to\infty} 2^y = 0$, then $I = \frac{1}{\ln 2}$

Hence improper integral is convergent and converges to $\frac{1}{\ln 2}$

Hence option (3) is correct.

8. **Explanation:**

$f(x) = x^2 \forall x \in [0,1]$

Therefore consider the interval $I_k = \left[\frac{k-1}{n}, \frac{k}{n}\right]$ for $k \in \mathbb{N}$

Then, $M(f, I_k) = \max f(x), x \in I_k$

$= \left(\frac{k}{n}\right)^2 (as\ I_k \subset [0,1])$

And, $m(f, I_k) = \min f(x), x \in I_k$

$= \left(\frac{k-1}{n}\right)^2 (as\ I_k \subset [0,1])$

Therefore, $\beta_n = L(f,P_n) = \sum_{k=1}^n m(f,I_k)(\text{length of } I_k)$

$= \sum_{k=1}^n \left(\frac{k-1}{n}\right)^2 \cdot \frac{1}{n}$

$= \frac{1}{n^3}\sum_{k=1}^n (k-1)^2$

$= \frac{1}{n^3}\frac{(n-1)(n)(2n-2+1)}{6}$

$= \frac{(n+1)(2n+1)}{6n^2}$

$\left(\lim_{n\to\infty}\alpha_n = \frac{1}{3} = \lim_{n\to\infty}\beta_n\right)$

Hence option (3) is correct.

9. **Explanation:** Let $I = \int_0^{\pi/2} \log\sin x dx$

$I = \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right) dx$

Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$

$= \int_0^{\pi/2} \log\cos x dx$

Therefore $2I = I + I = \int_0^{\pi/2} (\log\sin x + \log\cos x)\,dx$

$= \int_0^{\pi/2} \log(\sin x.\cos x)dx$

$= \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$

Let, $I_1 = \int_0^{\pi/2} \log\sin 2x dx$

Let, $t = 2x$, then $dt = 2dx$

$\Rightarrow I_1 = \frac{1}{2}\int_0^\pi \log\sin t dt = \int_0^{\pi/2} \log\sin t dt$ and $\int_0^{\pi/2} \log 2 dx = \frac{\pi}{2}\log 2$

$\therefore 2I = I - \frac{\pi}{2}\log 2$

i.e. $I = -\frac{\pi}{2}\log 2$

Hence option (3) incorrect

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Unit 1.7: Differentiability

[CUET- PG]

[CUET PG 2022]

1. Rolle's theorem is applicable for
(1) $f(x) = x^2$ *in* $[-1, 0]$.
(2) $f(x) = x(x+3)e^{-x/2}$ *in* $[-3,0]$
(3) $f(x) = \{x \sin \sin \frac{1}{x}$ *if* $x \neq 0\ 0$ *if* $x = 0$, if $\left[-\frac{1}{\pi}, \frac{1}{\pi}\right]$
(4) $f(x) = |x|$ *in* $[-1,1]$.

[CUET PG 2022]

2. Match List I with List II

List I	List II
f(x) in the interval	'c' in Rolle's Theorem, f'(c) = 0
A. $(x-1)(x-2)(x-3)$ in $[1,3]$	I. $\frac{2m-1}{2(m+n)-1}a$
B. $\sin \sin x$ in $[0, \pi]$	II. $\frac{n}{m+n}a$
C. $x^{2m-1}(a-x)^{2n}$ in $[0, a]$	III. $2 - \frac{2}{\sqrt{3}}$
D. $x^{2n}(a-x)^{2m}$ in $[0, a]$	IV. $\frac{\pi}{2}$

Choose the correct answer from the options given below:
(1) A-I, B-II, C-IV, D-III
(2) A-II, B-III, C-I, D-IV
(3) A-III, B-IV, C-I, D-II
(4) A-III, B-IV, C-II, D-I

[CUET PG 2022]

3. Given below are two statements:
Statement I:
Consider $f : R^2 \to R$ defined by $f(x, y) := x^2$. Then f has a local extremum at (0, 0).
Statement II:
Consider $f : R^2 \to R$ defined by $f(x, y) := xy$. Then f has a saddle point at (0, 0).
In the light of the above statements, choose the correct answer from the options given below
(1) Both statement I and statement II are true
(2) Both statement I and statement II are false
(3) Statement I is true but statement II is false
(4) Statement I is false but Statement II is true

[CUET PG 2022]

4. Given below are two statements:
Statement I:
$\frac{1}{x+y}$ is a homogeneous function of degree - 1.
Statement II:
A function $f(x, y)$ is said to be a homogeneous function of degree n in x and y if it satisfies $f(x, y) = x^n g(x)$
In the light of the above statements, choose the correct answer from the options given below
(1) Both statement I and statement II are correct
(2) Both statement I and statement II are incorrect
(3) Statement I is correct but statement II is incorrect
(4) Statement I is incorrect but statement II is correct

[CUET PG 2023]

5. If $f(x)$ satisfies the conditions of Rolle's theorem in [1, 2] and $f(x)$ is continuous in [1, 2], then $\int_1^2 f'(x)\,dx$ is equal to
(1) 4
(2) 0
(3) 1
(4) 2

[CUET PG 2023]

6. If f is twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2$ and $h(5) = 11$, then $h(10) =$
(1) 22
(2) 11
(3) 15
(4) 21

[CUET PG 2023]

7. If $P(x)$ is a polynomial of degree 10 with leading coefficient as 11 and having $(x-1), (x-2), (x-3) \ldots (x-10)$ as factor, then the coefficient of x^9 in $P(x)$ is
(1) – 55
(2) 605
(3) – 605
(4) 55

Answer Key						
1	2	3	4	5	6	7
2	3	4	1	4	2	3

[CU-CET]

[CUCET 2016]

1. Let $f : (1, \infty) \to R$ be a function defined by $f(x) = 2$. Then the derivative of f is
(1) $\frac{1}{x \log \log x} f(x)$
(2) $-\frac{1}{x \log \log x} f(x)$
(3) $\frac{1}{\log \log x} f(x)$
(4) $-\frac{1}{\log \log x} f(x)$

[CUCET 2016]

2. Let $f : R \to R$ be a function defined by $f(x) = \{ax+, b, if\ x \leq 1\ ax^2 + c, if\ 1 < x \leq 2\ \frac{dx^2+1}{x}, if\ x >$ 2 where a, b, c, d are constants. The values of a, b, c, d so that f is differentiable on R, are

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(1) $a = 0, b = c = 1, d = \frac{1}{4}$

(2) $a = 0, b = c = -1, d = \frac{1}{2}$

(3) $a = 0, b = c = -1, d = \frac{1}{4}$

(4) $a = -1, b = c = 0, d = \frac{1}{2}$

[CUCET 2017]

3. The derivative of a periodic function with period t is

(1) a constant function

(2) a periodic function with period t

(3) a non-periodic function

(4) none of the above

[CUCET 2017]

4. Using Rolle's theorem, the equation $a_0x^n + a_1x^{n-1} + \cdots + a_n = 0$ has atleast one root between 0 and 1, if

(1) $\frac{a_0}{n} + \frac{a_1}{n-1} + \cdots + a_{n-1} = 0$

(2) $\frac{a_0}{n-1} + \frac{a_1}{n-2} + \cdots + a_{n-2} = 0$

(3) $\frac{a_0}{n+1} + \frac{a_1}{n} + \cdots + a_n = 0$

(4) $na_0 + (n-1)a_1 + \cdots + a_{n-1} = 0$

[CUCET 2017]

5. The set of points where $f(x) = |sinx|$ is not differentiable is

(1) empty

(2) $\{0\}$

(3) $\{k\pi; k \in Z\}$

(4) $\left\{\frac{k\pi}{2}; k \in Z\right\}$

[CUCET 2017]

6. Let $P_n(x)$ be a Taylor's polynomial of degree $n \geq 0$ for the function e^x about 0. Then, the error in this approximation is

(1) $\frac{x^n}{n!}e^t$ for some $t, 0 < t < x$

(2) $\frac{x^n}{(n+1)!}e^t$ for some $t, 0 < t < x$

(3) $\frac{x^{n+1}}{n!}e^t$ for some $t, 0 < t < x$

(4) $\frac{x^{n+1}}{(n+1)!}e^t$ for some $t, 0 < t < X$

[CUCET 2017]

7. If $f(x) = (x + |x|)\,|x|$, for all $x \in R$, then which of the following is incorrect?

(1) f is continuous

(2) f is not differentiable for some x

(3) f' is continuous

(4) f' is differentiable

[CUCET 2017]

8. The Maclaurin's expansion of $tan\ tan\ x$ in powers of x is valid in the interval

(1) $(-\infty, \infty)$

(2) $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

(3) $(-\pi, \pi)$

(4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

[CUCET 2019]

9. If Rolle's theorem holds for $f(x) = x^2 + ax^2 + bx$ on $[-2,2]$ at $x = 1$, then

(1) $a = 1/2, b = -4$

(2) $a = 2, b = 4$

(3) $a = -1/2, b = 4$

(4) $a = 4, b = 1/2$

[CUCET 2019]

10. The local maxima of $x^3 - 3x + 3$ is attend at

(1) $x = -1$

(2) $x = 1$

(3) $x = 0$

(4) $x = 3$

[CUCET 2019]

11. The function $f(x) = sin\ 3x, x \in [0, \pi/2]$ is increasing in the interval

(1) $(0, \pi/6)$

(2) $(\pi/6, \pi/2)$

(3) $(0, \pi/2)$

(4) $(\pi/3, \pi/2)$

[CUCET 2019]

12. The function $f(x) = x^2$ is not uniformly continuous on the interval

(1) $[1, 1]$

(2) $[1, 2]$

(3) $[0, \infty)$

(4) $[0, 1]$

[CUCET 2020]

13. If $f(1) = 4$ and $f'(1) = 2$, then the value of the derivative of the function $log\ log\ (f(e^x))$ w.r.t x at $x = 0$ is

(1) $\frac{1}{2}$

(2) 1

(3) 2

(4) 4

[CUCET 2020]

14. Let $f(x) = e^x$ and $g(x) = e^{-x}$, for all $x \in [a, b]$. Then the value of the point $c \in (a, b)$ of the Cauchy's mean value theorem is

(1) ab

(2) $\frac{a+b}{2}$

(3) $\frac{2ab}{a+b}$

(4) $\sqrt{ab}$

[CUCET 2020]

15. The coefficient of $(x-1)^2$ in the Taylor series expansion of the function $f(x) = xe^x, x \in R$ about the point $x = 1$ is

(1) $\frac{e}{2}$

(2) $\frac{3e}{2}$

(3) $2e$

(4) $3e$

[CUCET 2021]

16. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

(1) $\frac{1}{256}$

(2) $\frac{2^4}{256}$

(3) $\frac{e^4}{256}$

(4) $\frac{e^2}{256}$

[CUCET 2021]

17. Which one of the following is incorrect?

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(1) If a real valued function f is monotonic on $[a, b]$, then it is integrable on $[a, b]$

(2) A real valued bounded function f, having a finite number of points of discontinuity on $[a, b]$, is not integrable on $[a, b]$

(3) If a real valued function f is integrable on $[a, b]$, then f^2 is also integrable on $[a, b]$

(4) Every real valued continuous function f on $[a, b]$ is integrable on $[a, b]$

Answer Key									
1	2	3	4	5	6	7	8	9	10
2	1	2	3	3	1	4	4	1	1
11	12	13	14	15	16	17			
1	3	1	2	2	4	2			

[PRACTICE QUESTIONS]

1. The real-valued functions f, ϕ, ψ are derivable in [a, b], then there exists at least one $c \in (a, b)$ such that

(1) $\begin{vmatrix} f'(a) & \phi'(a) & \psi'(a) \\ f(b) & \phi(b) & \psi(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$

(2) $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$

(3) $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f'(b) & \phi'(b) & \psi'(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$

(4) $\begin{vmatrix} f'(a) & \phi'(a) & \psi'(a) \\ f'(b) & \phi'(b) & \psi'(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$

2. With the help of mean value theorem, for $x > 0, 0 < \theta < 1, \log_{10}(x + 1)$ can be expressed as

(1) $\frac{x \log_{10} e}{1+\theta x}$ (2) $\frac{x}{1+\theta x}$

(3) $\frac{\theta x}{1+x}$ (4) $\frac{\theta x}{[1+\theta x]}$

3. If $y = \log\left(x + \sqrt{1 + x^2}\right)$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to:

(1) 2 (2) 1

(3) -1 (4) 0

4. If $y = \tan^{-1} x$, then $(1 + x^2)\frac{d^2y}{dx^2}$ is equal to :

(1) $\frac{dy}{dx}$ (2) $2x\frac{dy}{dx}$

(3) $x\frac{dy}{dx}$ (4) $-2x\frac{dy}{dx}$

5. If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y$ is equal to :

(1) -1 (2) 1

(3) 0 (4) None of these

6. If $y = \left(\log\left(x + \sqrt{1 + x^2}\right)\right)^2, (y_n)_{(0)} = \frac{d^2y}{dx^2}$ at, $x = 0, n \geq 1$, then $(y_{n+2})_{(0)} + n^2(y_n)_{(0)}$ is :

(1) -1 (2) 0

(3) 1 (4) None of these

7. If $y^{1/m} + y^{-1/m} = 2x, y_n = \frac{d^n y}{dx^n}$, then $(x^2 - 1)y_{n+2} + (2n - 1)xy_{n+1}$ is equal to :

(1) $(m^2 - n^2)y_n$ (2) $(m^2 + n^2)y_n$

(3) $m^2 y_n$ (4) $n^2 y_n$

8. If $f(x)$ is real - valued function such that $|f(x)| \leq |x|^{\frac{3}{2}}$ for $-1 < x < 1$, then :

(1) f is differentiable at $x = 0$ and f'(0) = 0

(2) f is not differentiable at $x = 0$

(3) f is differentiable at $x = 0$ and $f'(0) = 1/2$

(4) f is differentiable at $x = 0$ and $f'(0) = 1$

9. If $f(x) = \begin{cases} \sin 2x, & x \leq 0 \\ mx, & x > 0 \end{cases}$, then f is differentiable at $x = 0$ if :

(1) $m = 0$ (2) $m = 1$

(3) $m = 2$ (4) $m = 3$

10. If $u = \sin\left(\frac{x}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(1) 0 (2) 1

(3) 2 (4) -1

11. If $u(x, y)$ is a homogeneous function of degree 2, then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ is

(1) u (2) $2u$

(3) 0 (4) $3u$

12. If $x = r\cos\theta, y = r\sin\theta$ and n is a positive integer, then $\frac{\partial^2}{\partial x\, \partial y}(r^n \cos n\theta)$

(1) $-n(n + 1)r^{n-2}\sin(n - 2)\theta$

(2) $n(n - 1)r^{n-2}$

(3) $n(n - 1)r^{n-2}\sin(n - 2)\theta$

(4) $\sin(n - 2)\theta$

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13. If $\theta = t^n e^{-r^2/4t}$ and $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$, then the value of n is

(1) 3 (2) $\frac{3}{2}$

(3) $-\frac{3}{2}$ (4) 1

14. If $f(x,y,z) = \log(x^3+y^3+z^3-3xyz)$, then $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 f$ is.

(1) $\frac{3}{(x+y+z)^2}$ (2) $-\frac{3}{(x+y+z)^2}$

(3) $\frac{9}{(x+y+z)^2}$ (4) $-\frac{9}{(x+y+z)^2}$

15. The nth derivative of $(x-1)^n$ with respect to x is

(1) nx (2) $n(x-1)$

(3) $n1$ (4) n

16. When transformed to polar coordinates, the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ becomes

(1) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$

(2) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$

(3) $\frac{\partial^2 u}{\partial r^2}+\frac{2}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta}=0$

(4) $\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial \theta^2}=0$

17. If u is a homogeneous function of x and y of degree n, then the value of $x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x\,\partial y}$ is

(1) $(n-1)\frac{\partial u}{\partial x}$ (2) $n\frac{\partial u}{\partial x}$

(3) $(n-1)\frac{\partial u}{\partial y}$ (4) $n\frac{\partial u}{\partial y}$

18. Writing the mean value theorem as $f(b)-f(a)=(b-a)f'(c), a<c<b$, the value of c, if $f(x)=x(x-2), a=0, b=1$, is

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{2}{5}$ (4) $\frac{2}{3}$

19. If $r=\sqrt{a^2+b^2}$ and $\phi=\tan^{-1}(b/a)$, then the nth derivative of $e^{ax}\cos(bx+c)$ is

(1) $r^n e^{ax}\sin(bx+c+n\phi)$

(2) $r^n e^{ax}\cos(bx+c+n\phi)$

(3) $r\,e^{ax}\sin(bx+c+n\phi)$

(4) $r\,e^{ax}\cos(bx+c+n\phi)$

20. If $g:\mathbb{R}\to\mathbb{R}$ be defined by $g(x)=\begin{cases}\frac{\sin x}{x}, & if\ x\neq 0\\ 1, & if\ x=0\end{cases}$ then

(1) g is not continuous.

(2) g is continuous but not differentiable.

(3) g is differentiable.

(4) g is not bounded

21. Let $f:[-1,1]\to\mathbb{R}$ be defined by $f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}$ and F be its indefinite integral

Which of the following is not true?

(1) $F'(0)$ does not exist

(2) F is an anti-derivative of f on [-1, 1]

(3) F is Riemann integrable on [-1,1]

(4) F is continuous on [-1,1]

22. Let $n\in\mathbb{N}, n\geq 3$, be fixed and let $f:[0,1]\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}x, & 0\leq x\leq 1/n\\ x-\frac{(2k-1)}{2n}, & \frac{k-1}{n}<x\leq\frac{k}{n}\\ & k=2,3\ldots.,n\end{cases}$$

Then

(1) f is continuous and Riemann integrable on [0,1]

(2) f is not continuous but is Riemann integrable on [0, 1]

(3) f is continuous but not Riemann integrable on [0, 1]

(4) f is neither continous nor Riemann integrable on [0, 1].

23. If $f:[0,10]\to\mathbb{R}$ is defined as

$$f(x)=\begin{cases}0, & 0\leq x<2,\\ 1, & 2\leq x\leq 5,\\ 0, & 5<x\leq 10,\end{cases}$$

and $F(x)=\int_0^x f(t)dt$ then

(1) $F(x)=3$ for $x\leq 5$

(2) $F'(x)=f(x)$ for every x.

(3) F is not differentiable at $x=2$ and $x=5$

(4) F is differentiable everywhere on $[0,10]$

24. Consider the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}\sin x & x\geq 0\\ 1-\cos x, & x<0\end{cases}$

Which of the following statements is true about f?

(1) f differentiable.

(2) f' is contirurorrs but NOT.diffrentiable

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(3) f is discontinuous.
(4) none of the above statements is true.

25. Let $f:(-1,1)\to\mathbb{R}$ be a differentiable function. Which one of the following is necessarily true?
(1) If $f'(0)>0$, then $f(x)>f(0)$ for all $x\in(0,1)$.
(2) If $f'(0)<0$, then $f(x)<f(0)$ for all $x\in(0,\delta)$ for some $\delta>0$.
(3) If $f'(0)=0$, then $f(x)=f(0)$ for all $x\in(-\delta,\delta)$ for some $\delta>0$.
(4) If $f'(0)>0$,then $f(0)>0$

26. Compute $\frac{dw}{dt}\Big|t=\frac{x}{4}$ *where* $w=xy^2z, x=\cos(t), y=\sin(t), z=t$
(1) 1
(2) $(\pi+4)/(8\sqrt{2})$
(3) $\frac{1}{(2\sqrt{2})}$
(4) $(\pi+1)/(2\sqrt{2})$

27. Let n be a positive integer. Let $f(x)=x^{n+2}\sin\frac{1}{x}$ if $x\neq 0$ and let $f(0)=0$. For what value of n will f be twice differentiable but with its second derivative discontinuous at $x=0$?
(1) $n<-1$
(2) $n>+1$
(3) $n<+1$
(4) $n>-1$

28. What is the coefficient of x^8 in the expansion of $x^2\cos(x^2)$ around $x=0$?
(1) 0
(2) 1
(3) 2
(4) 3

29. Let $g:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $|g'(x)|\leq M$ for all $x\in\mathbb{R}$. For what values of ϵ will the function $f(x)=x+\epsilon g(x)$ be necessarily one-to-one?
(1) $f'(x)=1-\epsilon\, g'(x)$
(2) $f(x)=1+\epsilon\, g(x)$
(3) $f'(x)=1+\epsilon\, g(x)$
(4) $f'(x)=1+\epsilon\, g'(x)$

30. Differentiate:
$f(x)=\int_x^{x^2}e^{t^2}dt, x>1.$
(1) $e^{x^4}(1+2x)-2e^{x^2}$
(2) $e^{x^4}(1+2x)+2e^{x^2}$
(3) $e^{x^4}(1+4x)-2e^{x^2}$
(4) $e^{x^4}(1+2x)-2e^{x}$

31. Let $f:[-1,2]\to\mathbb{R}$ be given by $f(x)=2x^3-x^4-10$. What is the value of x where f assumes its minimum value?
(1) 8.313
(2) -8.313
(3) -6.313
(4) 6.313

32. Let $x>0$. Define
$f(x)=\int_0^x\frac{\sin xy}{y}dy.$
Evaluate $f'(x)$ as a function of x.
(1) $\frac{2sinx^2}{x}$
(2) $2sinx^2$
(3) $\frac{2sin}{x}$
(4) $\frac{4sinx^2}{x}$

33. Which of the following functions is not differentiable at $x=0$?
(1) $f(x)=\begin{cases}x^2, & if\ x\ is\ rational\\ 0, & if\ x\ is\ irrational.\end{cases}$
(2) $f(x)=|x|x.$
(3) $f(x)=\begin{cases}x^2\sin\frac{1}{x} & if\ x\neq 0\\ 0 & if\ x=0\end{cases}$
(4) None of the above

34. Find the coefficient of x^7 in the Maclaurin series expansion of the function $f(x)=\sin^{-1}x$.
(1) $\frac{1.3.5}{2.4.6.8}$
(2) $\frac{3.5.7}{2.4.6.7}$
(3) $\frac{1.3.5}{4.6.7}$
(4) $\frac{1.3.5}{2.4.6.7}$

35. Which of the following maps is not differentiable everywhere?
(1) $f(x)=|x|^3x, x\in\mathbb{R}.$
(2) $f:\mathbb{R}\to\mathbb{R}$ such that $|f(x)-f(y)|\leq|x-y|^{\sqrt{2}}$ for all x and $y\in\mathbb{R}$.
(3) $f(x)=x^3\sin\frac{1}{\sqrt{|x|}}$ when $x\neq 0$ and $f(0)=0.$
(4) None of the above

36. Write down the coefficient of x^7 in the Taylor series expansion of the function $f(x)=\log(x+\sqrt{1+x^2})$ about the origin.
(1) $\frac{5}{112}$
(2) $\frac{-5}{112}$
(3) $\frac{-5}{114}$
(4) $\frac{5}{114}$

38. In each of the following, verify if the given function $f:\mathbb{R}\to\mathbb{R}$ is continuously differentiable, differentiable but not continuously differentiable or not differentiable at the origin:
(1) $f(x)=x\sin\frac{1}{\sqrt{x}}$, if $x\neq 0$ and $f(0)=0.$
(2) $f(x)=$ [illegible]

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(3) $f(x) = x\sin|x|$

(4) None of the above

39. Pick out the false statements:

(1) $|\cos^2 x - \cos^2 y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

(2) If $f : \mathbb{R} \to \mathbb{R}$ satisfies

$|f(x) - f(y)| \leq |x - y|^{\sqrt{2}}$

for all $x, y \in \mathbb{R}$, then f must be a constant function.

(3) Let $f : \mathbb{R} \to \mathbb{R}$ be continuously differentiable and such that $|f'(x)| \leq \frac{4}{5}$

for all $x \in \mathbb{R}$. Then, there exists a unique $x \in \mathbb{R}$ such that $f(x) = x$.

(4) None of the above

40. Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is differentiable at $x = a$. Evaluate the following:

$\lim_{x \to a} \frac{a^n f(x) - x^n f(a)}{x - a}$.

(1) $a^n(f'(a) - f(a)a^n \log a)$

(2) $a^n(f'(a) + f(a)a^n \log a)$

(3) $a^n(f(a) - f(a)a^n \log a)$

(4) $a^n(f(a) - f(a)a \log a)$

41. Write down explicitly the expression for the n-th derivative of the function $f(x) = x^2e^{3x}$.

(1) $f^n(x) = 3^{n-2}e^{3x}[9x^2 + 6nx + n(n+1)]$

(2) $f^n(x) = 3^{n-2}e^{3x}[9x^2 - 6nx + n(n-1)]$

(3) $f^n(x) = 3^{n-2}e^{3x}[9x^2 + 6nx + n(n-1)]$

(4) $f^n(x) = 3^{n-2}e^{3x}[9x^2 - 6nx + n(n+1)]$

42. Let $f :]0, \infty[\to]0, \infty[$ be differentiable at $a > 0$. Evaluate:

$\lim_{x \to a} \left(\frac{f(x)}{f(a)}\right)^{\frac{1}{\log x - \log a}}$.

(1) $e^{\frac{aF(a)}{F(a)}}$ (2) $e^{\frac{F(a)}{F(a)}}$

(3) $e^{\frac{aF}{F}}$ (4) $e^{\frac{aF(a)}{aF(a)}}$

43. Define

$f(x) = \begin{cases} x^2 & \text{if } x \in]0,2[\text{ and } x \text{ is rational} \\ 2x - 1 & \text{if } x \in]0,2[\text{ and } x \text{ is irrational} \end{cases}$

Which of the following statements are true?

(1) f is not differentiable at $x = 1$.

(2) f is differentiable at $x = 1$.

(3) f is differentiable only at $x = 1$.

(4) None of the above

44. Write down the Taylor series expansion about the origin valid in the interval $]-1,1[$ for the function $f(x) = \log\sqrt{1+x^2}$.

(1) $log(\sqrt{1+x^2}) = \sum_{n=1}^{\infty} \frac{(-1)^1}{n} x^{2n}$

(2) $log(\sqrt{1+x^2}) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{2n}$

(3) $log(\sqrt{1+x^2}) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

(4) $log(\sqrt{1+x}) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{2n}$

45. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable at $x = a$ and let $f(a) > 0$. Evaluate:

$\lim_{x \to a} \left(\frac{f(x)}{f(a)}\right)^{\frac{1}{\log x - \log a}}$

(1) $e^{\frac{aF(a)}{F(a)}}$ (2) $e^{\frac{F(a)}{F(a)}}$

(3) $e^{\frac{aF}{F}}$ (4) $e^{\frac{aF(a)}{aF(a)}}$

46. Which of the following functions is not differentiable at $x = 0$?

(1) $f(x) = \begin{cases} \tan^{-1}\left(\frac{1}{|x|}\right) & \text{if } x \neq 0 \\ \frac{\pi}{2}, & \text{if } x = 0 \end{cases}$

(2) $f(x) = |x|^{\frac{1}{2}}x$.

(3) $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

(4) None of the above

47. For what values of $x \in \mathbb{R}$ is the following function decreasing?

$f(x) = 2x^3 - 9x^2 + 12x + 4$.

(1) (1,-2) (2) (-1,-2)

(3) (1,2) (4) (-1,2)

48. Which of the following functions defined on $\mathbb{R}$ is not differentiable?

(1) $f(x) = x|x|$.

(2) $f(x) = [x]\sin^2 \pi x$.

(3) $f(x) = \sqrt{|x|}$.

(4) None of the above

(The symbol $[x]$ in statement (b) stands for the integral part of $a \in \mathbb{R}$, i.e. the largest integer less than, or equal to, x.)

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49. Which of the following statements are true?
(1) Let f be continuously differentiable in $[a,b]$ and twice differentiable on $]a,b[$. If $f(a)=f(b)$ and if $f'(a)=0$, then there exists $x_0 \in]a,b[$ such that $f''(x_0)=0$.
(2) Let f be continuously differentiable in $[a,b]$. If $f(a)=f(b)$ and if $f'(a)=f'(b)$, then there exists x_1 and x_2 in $]a,b[$ such that $x_1 \neq x_2$ and such that $f'(x_1)=f'(x_2)$.
(3) Let f be continuously differentiable on $[0,2]$, and twice differentiable on $]0,2[$. If $f(0)=0, f(1)=1$ and $f(2)=2$, then there exists $x_0 \in]0,2[$ such that $f''(0)=0$
(4) None of the above

50. Let $f(x)=|x|^{\frac{5}{2}}, x \in \mathbb{R}$ R. Which of the following statements are true?
(1) The function f is not differentiable at $x=0$.
(2) The function f is differentiable everywhere and $f'(x)=\frac{5}{2}|x|^{\frac{3}{2}}$.
(3) The function f is differentiable everywhere and $f'(x)=\frac{5}{2}|x|^{\frac{1}{2}}x$.
(4) None of the above

51. Which of the following functions are uniformly continuous over $\mathbb{R}$?
(1) $f(x)=x^3$.
(2) $f(x)=\int_0^x g(t)dt$, where $g:\mathbb{R}\to\mathbb{R}$ is a continuously differentiable function.
(3) $f(x)=\int_0^x g(t)\,dt$, where $g:\mathbb{R}\to\mathbb{R}$ is a continuous function which is bounded.
(4) None of the above

52. Evaluate:
$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x}dx$.
(1) $\frac{\log(\sqrt{2}+1)}{\sqrt{2}}$
(2) $\frac{\log(2+1)}{\sqrt{2}}$
(3) $\frac{\log(1+1)}{\sqrt{2}}$
(4) $\frac{\log(\sqrt{2}-1)}{\sqrt{2}}$

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Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
4	2	4	4	3	2	1	1	1	1
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2	3	3	4	3	2	1	1	2	3
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
2	2	3	2	2	1	1	1	4	1
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
2	1	3	4	4	2	-	1	3	1
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
3	1	1	2	1	4	4	4	1	3
51	**52**								
1	1								

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EXPLANATIONS

[CUET- PG]

1. **Explanation:**

Roll's the continuous on $[a,b]$ & differentiable on (a,b) such that

$F(a)=f(b)$ then $f'(x)=0\ x\in[a,b]$

1. $f(x)=x^2\ \ x\in[1,0]$

$f(a)=f(-1)=.1$

$f(a)=f(0)=0$

$f(a)\neq f(b)$

Roll's theorem is not applicable

2. $F(x)=x(x+3)e^{-\frac{x}{2}}\ x\in[-3,0]$

$F(-3)=0\ \ F(0)=0$

$F(-3)=f(0)$

$F(x)$ is differentiable on $(-3,0)$

⇒ Roll's theorem is applicable

3. $F(x)=\begin{cases}x\sin\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$ in $\left(\frac{-1}{\pi},\frac{1}{\pi}\right)$

$f(x)$ is continuous

$F\left(-\frac{1}{\pi}\right)=0=f\left(\frac{1}{\pi}\right)$

But $f'(x)$ is not exists at $x=0$

⇒ Roll's theorem is not applicable.

4. $F(x)=|x|$ in $[-1,1]$

$F(-1)=F(1)$

$F(x)$ is continuous on $[-1,1]$

But not, differentiable on $(-1,1)$

⇒ Roll's theorem is not applicable.

2. **Explanation:**

(A) $F(x)=(x-1)(x-2)(x-3)$ in $[1,3]$

$f(1)-f(3)=0$

$f(x)$ is continuous on $[1,3]$ & differentiable $(1,3)$

So by Roll's theorem

$\exists\subset\in(1,3)$ such

That

$F'(c)=0$

$F'(C)=3c^2-12C+11=0$

$C=\frac{12\pm\sqrt{144-4\times11}}{2\times3}$

$C=\frac{12\pm\sqrt{144-132}}{6}$

$C=\frac{12\pm\sqrt{12}}{6}$

$C=2\pm\frac{\sqrt{12}}{6}$

$C=2\pm\frac{1}{\sqrt{6}}$

(B) $F(x)=\sin x,\qquad [0,\pi]$

$F(0)=f(\pi)$

& $F(x)$ is continuous & differential on $(0,\pi)$

By Roll's theorem

$F'(C)=0$

$\cos(C)=0$

$C=\frac{\pi}{2}$

(C) $f(x)=x^2(a-x)^{2m}$

$f(0)=F(a)=0$

By Roll's theorms

$\exists\ C\in[0,0]$

Such that

$F'(C)=1$

$2n\,C^{2n-1}(a-c)^{2m}-2mc^{2n}\,(a-c)^{2n-1}=0$

$2n(a-c)-2mc=0$

$2n(a-c)-2mc=0$

$2n(a-c)=2mc$

$2na-2mc=2mc$

$C=\frac{2na}{m+n}$

(D) $F(x)=x^{2m-1}\,(a-x)^{2n}$

$F'(C)=C^{2m-1}\,(2m-1)\,(a-c)^{2n}-2n\,C^{2m-1}\,(a-c)^{2n-1}$

$C^{2m-2}\,(2m-1)(a-c)^{2n}-2nC^{2m-1}(a-c)^{2n-1}=0$

$2m-1(a-c)=2nc$

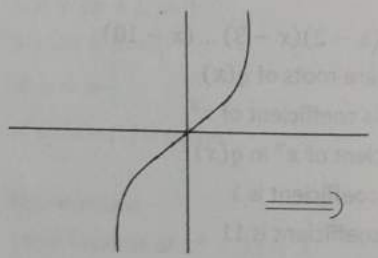
$(2m-1)a=(2n\perp 2m-1)C$

$C=\frac{(2m-1)a}{2(m+n)-1}$

3. **Explanation:**

I $f(x,y)=x^3$

In the Nbd of $(0,0)$



This is redqular increasing or decreasing

⇒ It does not have maxima

II. $F(x,y)=xy$

$F_x=y$ $\qquad F_y=x$

$F_y=1$ $\qquad F_yx=1$

$F_{yy}=0$

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$H = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$|H| = -1 < 0 \Rightarrow$ saddle point.

4. Explanation:

I. $F(x) = \frac{1}{x+y}$

$F(tx) = \frac{1}{tx+ty} = t^{-1}\left(\frac{1}{x+y}\right) = t^{-1}F(x,y)$

Statements I is true.

II. $F(x,y)$ is Homogeneous function of degree 'n' if it satisfies

$F(tx,ty) = t^n F(x,y)$

So statement II wrong.

5. Explanation:

If $f(x)$ satisfies the conditions of Roll's theorem in $[1,2]$

$f(1) = f(2)$

$\int_1^2 f'(x)\,dx = f(2) - f(1) = 0$

6. Explanation:

$f''(x) = -f(x)$ & $f'(x) = g(x)$

$h(x) = [f(x)]^2 + [g(x)]^2$ $\qquad h(5) = 11$

$h(5) = [f(5)]^2 + [g(5)]^2 = 11$

$f'(x) = g(x)$

$f''(x) = g'(x)$

$g'(x) = -f(x)$

$h(x) = [f(x)]^2 + [g(x)]^2$

$h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$h'(x) = 2f(x) \times g(x) + 2g(x) \times f(x)$

$h'(x) = 0$

$\Rightarrow h(x)$ is constant

$h(10) = 11$

7. Explanation:

$q(x) = (x-1)(x-2)(x-3)\ldots(x-10)$

$\Rightarrow$ 1,2,3,4, ... 10 are roots of $q(x)$

So sum of roots is coefficient of x^9

$\Rightarrow -55$ is coefficient of x^9 in $q(x)$

$q(x)$ ha leading coefficient is 1

In $P(x)$ leading coefficient is 11

So $P(x) = 11q(x)$

$\Rightarrow -605$ is coefficient of x^9 in $P(x)$

[CU-CET]

1. Explanation:

$f: (1,\infty) \to R$ be function given by

$f(x) = \log_x 2$

$= \frac{d}{dx}\left(\frac{\log 2}{\log x}\right)$ (change of base law)

$= \frac{\log x(0) - \log 2\left(\frac{1}{x}\right)}{(\log x)^2} = \frac{-\frac{\log 2}{x}}{(\log x)^2}$

$= \frac{-\log 2}{x} \times \frac{1}{(\log x)^2} = -\frac{\log 2}{x \log x}\left(\frac{1}{\log x}\right)$

$= \frac{-\log_x 2}{\log x} = \frac{-f(x)}{\log x}$

$\Rightarrow$ option (4) is correct

2. Explanation:

$$f(x) = \begin{cases} ax + b & if\ x \le 1 \\ ax^2 + c & if\ 1 < x \le 2 \\ \frac{dx^2+1}{x} & if\ x \ge 2 \end{cases}$$

Given function is differentiable

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = \lim_{x\to 1^-} f(x)$

$\therefore \lim_{x\to 1^+} ax^2 + c = \lim_{x\to 1} ax + b = \lim_{x\to 1^-} ax + b$

$a + b = a + b \Rightarrow c = b$

lly, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2} f(x) = \lim_{x\to 2^-} f(x)$

$\lim_{x\to 2^+} \frac{dx^2+1}{x} = \lim_{x\to 2} ax^2 + c = \lim_{x\to 2^-} ax^2 + c$

$\frac{4d+1}{2} = 4a + c$

$4d + 1 = 8a + 2c$

$4d - 8a = 2c - 1$ (1)

$4d - 8a = 2b - 1$ (2)

Adding (1) & (2)

$8d = 4c - 2$ \pi

$8d = 2$

$d = {}^2/_8$

$d = {}^1/_4$

(take $c = b = 1$)

$\therefore 4d - 8a = 2c - 1$

$4\left(\frac{1}{4}\right) - 8a = 1$

$-8a = 1 - 1$

$-8a = 0$

$\Rightarrow a = 0$

$\therefore a = 0, b = c = 1, d = {}^1/_4$

Function f is differentiable on $\mathbb{R}$

$\Rightarrow$ option (1) is correct

3. Explanation:

The derivative of a periodic function with period t is periodic with period 3

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4. **Explanation:**

$a_0x^n + a_1x^{n-1} + \cdots + a_n = 0$ has atleast one root between 0 to 1.

Consider the function

$\phi(x) = \int (a_0x^n + a_1x^{n-1} + \cdots + a_n)\,dx$

$= \frac{a_0x^{n+1}}{n+1} + \frac{a_1x^n}{n} + \cdots + a_nx + A - (1)$

Where A is constant

$\phi(x)$ being algebraic poly is continuous in the closed interval $[0,1]$ & diff. on open interval $(0,1)$

If $f(0) = f(1)$ then we have

$\frac{a_0}{n+1} = \frac{a_1}{n} + \cdots + a_n = 0$ (2)

Thus $\phi(x)$ is given in (1) by satisfy all conditions of Rolle's then, so there exist atleast one $x = c$ s.t $0 < c < 1$

Where $\phi'(c) = 0$

I.e. atleast one root of $\phi'(c) = 0$

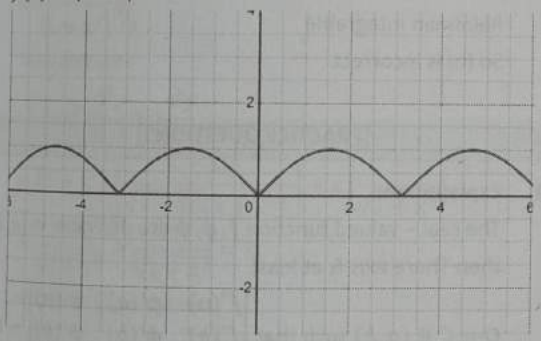
i.e. $a_1x^n + a_1x^{n+1} + \cdots + a_n = 0$ lie between 0 to 1

if equation (2) satisfied

⇒ option (C) satisfied

5. **Explanation:**

$f(x) = |\sin x|$



∴ set of points where $f(x) = |\sin x|$ is not differentiable is $\{k\pi, k \in \mathbb{Z}\}$

⇒ option (3) is correct

6. **Explanation:**

$P_n(x)$ be taylor poly. Degree $n \geq 0$

$f(x) = e^x$

The error of approximation

$f(x) = Tn(x) = \frac{1}{n!}, f^{n+1}(c)(x-a)^{n+1}$ for

Some c strictly between a and x

$\Rightarrow \approx \frac{x^n}{n!}e^t$ for some t $0 < t < x$

⇒ option (1) is correct

7. **Explanation:**

$f(x) = (x + |x|)|x|$

$f(x) = \begin{cases}(x+x)x & x > 0\\(x-x)x & x < 0\end{cases}$

$f(x) = \begin{cases}2x^2 & x > 0\\0 & x < 0\end{cases}$

$f'(x) = \begin{cases}4x & x > 0\\0 & x < 0\end{cases}$

⇒ option (4) is correct

8. **Explanation:**

$f(x,y) = \begin{cases}\frac{xy}{x^2+y^2} & x^2 + y^2 \neq 0\\0 & x + y^2 = 0\end{cases}$

We take path $y = mx$

$\lim_{x\to 0} \frac{xmx}{x^2+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^2+m^2x^2}$

$= \frac{m}{1+m^2}$

Limit depend on m.

Different limit along diff. value of m.

⇒ function is not continuous

⇒ function is not differentiable

⇒ option (3) is correct

9. **Explanation:**

$f(x) = x^3 + ax^2 + bx$ on $[-2,2]$ at $x = 1$ which holds Rolle's theorem.

∴ function is continuous as well as diff.

$f(a) = f(b)$ then $f'(x) = 0$ for some $a \leq x \leq b$

$f(-2) = -8 + a4 - 2b$ _____ (i)

$f(2) = 8 + 4a + 2b$ _____ (ii)

$\Rightarrow -8 + 4a - 2b = 8 + 4a + 2b - 16 = 4b$

$f'(x) = 0$

$f'(1) = 3x^2 + 2ax + b$

$\Rightarrow b = -4$

$= 3 + 2a + b = 0$

$3 + 2a + b = 0$

$\Rightarrow a = +\frac{1}{2}$

⇒ option (1) is correct

10. **Explanation:**

Local Maxima of $x^3 - 3x + 3$

$f(x) = x^3 - 3x + 3$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$

$f'(x) = 0$ $f''(x) = 6x$

$x = 1, -1$ $f''(1) = 6 > 0$ local minimize

$f(1) = (1)^3 - 3(1) + 3 = 1$ $f''(-1) = -6 < 0$

$f(-1) = (-1)^3 - 3(1) + 3 = 5$ *Local maxima*

∴ $x = -1$ attend local maxima.

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11. Explanation:

$f(x) = \sin 3x \quad x \in \left[0, \frac{\pi}{2}\right]$

CONSIDER $y = 3x$

$\Rightarrow y \in \left[0, \frac{\pi}{6}\right]$

12. Explanation:

$f(x) = x^2$ is uniformly continuous

At $[-1,1]$

$f(x) = x^2$

$f(-1) = 1$

$f(1) = 1$

at $[1,2]$

$f(x) = x^2$

$f(1) = 1$

$f(2) = 4$

At $[0,1]$

$f(x) = x^2$

$f(0) = 0$

$f(1) = 1$

But at $[0, \infty)$ $f(x) = x^2$ function undefined that interval.

$\Rightarrow$ option (3) is correct

13. Explanation:

Solution: Given: $f(1) = 4$ and $f'(1) = 2$

$f(x) = \log(f(e^x))$

$f'(x) = \frac{1}{f(e^x)} f'(e^x).e^x$

$f'(0) = \frac{1}{f(e^0)} f'(e^0).e^0$

$= \frac{1}{f(1)} f'(1).1 = \frac{1}{4}.2 = \frac{1}{2}$

14. Explanation:

Solution: Given $f(x) = e^x$ and $g(x) = e^{-x}, \forall x \in [a,b]$

Cauchy mean value theorem is

$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$

$\Rightarrow \frac{e^b-e^a}{e^{-b}-e^{-a}} = \frac{e^c}{-e^{-c}}$

$\Rightarrow \frac{e^b-e^a}{e^a-e^b} e^a e^b = -e^{2c}$

$\Rightarrow e^a e^b = e^{2c}$

$\Rightarrow e^{a+b} = e^{2c}$

$\Rightarrow a + b = 2c$

$\Rightarrow c = \frac{a+b}{2}, c \in (a,b)$

15. Explanation:

$f'(x) = e^x + x.e^x, f''(x) = e^x + e^{-x} + xe^x =$

$2e^x + xe^x$

$f''(1) = 2e + e = 3e$

And

$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$

$\therefore (x-1)^2 = \frac{f''(1)}{2!} = \frac{3e}{2}$

16. Explanation:

$F(x) = e^x \quad a = 2$

$F'(x) = e^x$

$F''(x) = e^x$

$F(x) \cong F(a) + \frac{F'(a)}{1!}(x-a) + \frac{F''(a)}{2!}(x-a)^2 +$

$\frac{F'''(a)}{3!}(x-a)^3 + \frac{f''''(a)}{4!}(x-a)^4 + \cdots$

$F'(x) = e^2 + e^2(x-a) + \frac{e^2}{2!}(x-2)^2 + \frac{e^2}{3!}(x-$

$2)^3 + \frac{e^2}{4!}(x-2)^4 + \cdots$

$\therefore coeff\ (x-2)^4$ is $\frac{e^2}{4!}$

17. Explanation:

f having a finite number of discontinuity then 'f' is Reimman integrable

So (b) is incorrect

[PRACTICE QUESTIONS]

1. Explanation:

The real – valued function f, ϕ, ψ are derivable in $[a,b]$ then There exis & at least

One $C \in (a,b)$ such that $\begin{vmatrix} f'(a) & \phi'(a) & \psi'(a) \\ f'(b) & \phi'(b) & \psi'(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$

$\Rightarrow$ option (4) is correct

2. Explanation:

For $x > 0\ 0 < \theta < 1\ \log_{10}(x+1) = f(x)$

$\frac{F(x)-F(0)}{x-0} = f'(\theta x)$

$\Rightarrow \frac{\log_{10}(1+x)}{x} = \frac{1}{1+\theta x}$

Now, $0 < \theta < 1\ x > 0 \Rightarrow \theta x < x$

$1 + \theta x < 1 + x$

$\frac{1}{1+\theta x} > \frac{1}{1+x}$

$\Rightarrow \frac{x}{1+\theta x} > \frac{x}{1+x}$ ——— (i)

Again $1 + \theta x > 1 \quad x > 0$

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$\frac{1}{1+\theta x} < 1$

$\frac{x}{1+\theta x} < x$ _____ (ii)

From (i) & (ii) we get

$\frac{x}{1+x} < \log_{10}(1+x) < x$

⇒ option (2) is correct

3. **Explanation:**

$y = \log\left(x + \frac{1}{\sqrt{1+x^2}}\right)$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}} \frac{d\{x+\sqrt{1+x^2}\}}{dx}$

$= \frac{1}{x+\sqrt{1+x^2}}\left\{1 + \frac{2x}{2\sqrt{1+x^2}}\right\}$

$= \frac{1}{x+\sqrt{1+x^2}} \times \frac{(\sqrt{1+x^2}+x)}{\sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

$\frac{d^2y}{dx^2} = \frac{\sqrt{1+x^2}(0) - (1)\left(\frac{1}{2}\frac{2x}{\sqrt{1+x}}\right)}{\left(\sqrt{1+x^2}\right)^2}$

$\frac{d^2y}{dx^2} = \frac{-x}{\sqrt{1+x^2}(1+x^2)}$

$= \frac{-x}{(1+x^2)^{3/2}}$

Now consider

$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$= (1+x^2)\left[\frac{-x}{\sqrt{1+x^2}(1+x^2)}\right] + x\left[\frac{1}{\sqrt{1+x^2}}\right]$

$= \frac{-x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} = 0$

⇒ option (4) is correct

4. **Explanation:**

let $y = \tan^{-1}(x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$

$\frac{d^2y}{dx^2} = \frac{-2x}{(1+x^2)^2}$

Now consider,

$(1+x^2)\frac{d^2y}{dx^2} = (1+x^2)\left(\frac{-2x}{(1+x^2)^2}\right)$

$= \frac{-2x}{1+x^2}$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} = -2x.\frac{dy}{dx}$

⇒ option (4) is correct

5. **Explanation:** we have,

$y = a\cos(\log x) + b\sin(\log x)$

$\Rightarrow \frac{dy}{dx} = \frac{-a\cos(\log x)}{x} + \frac{b\cos(\log x)}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{-a\cos(\log x) + b\cos(\log x)}{x}$

$\Rightarrow \frac{d^2y}{dx^2} = \left[\frac{-a\cos(\log x)}{x}\frac{b\sin(\log x)}{x}\right] - \frac{1(-a\sin(\log x)+b\cos(\log x))}{x^2}$

$= \frac{-a\cos(\log x) - b\sin(\log x) + a\sin(\log x) - b\cos(\log x)}{x^2}$

$= \frac{(a-b)\sin(\log x) - (a+b)\cos(\log x)}{x^2}$

Consider,

$x^2\frac{d^2y}{dx^2} + a\frac{dy}{dx} + y$

$= (a-b)\sin(\log x) - (a+b)\cos(\log x) + b\cos(\log x)$

$-a\sin(\log x) + a\cos(\log x) + b\sin(\log x)$

$= 0$

⇒ option (3) is correct

6. **Explanation:**

$y = \left(\log\left(x + \sqrt{1+x^2}\right)\right)^2$

$y' = \frac{2\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$

$\sqrt{1+x^2}y' = 2\log\left(x+\sqrt{1+x^2}\right)$

$(1+x^2)(y')^2 = 4y$

$(1+x^2)(2y'y'') + (2x)(y')^2 = 4y'$

$\Rightarrow (1+x^2)(2y'') + 2xy' = 4$

$\Rightarrow 2(1+x^2)y'' + 2xy' - 4 = 0$

∵ Lebnitz rules

$\left(y.F(x)\right)_n = {}^nL_0y_nF(x) + {}^nL_0y_{n+1}F'(x) + {}^nL_0y_0F^n(x)$

So,

$\{(1+x^2)y_2\}_n = {}^nL_0y_{n+2}(1+x^2) + {}^nL_0y_{n+1}(2x) + {}^nL_0y_n(2) + {}^nL_0y_{n-1}(0) + 0 \ldots + 0$

$= y_{n+2}(1+x^2) + ny_{n+1}(2x) + \frac{n(n-1)}{2}y_n(2)$

Now,

$(2xy_1)_n = {}^nL_0y_{n+1}(2x) + {}^nL_0y_n(2) + {}^nL_0y_{n-1}(0) + 0 +$

$(2xy_1)_n = y_{n+1}(2x) + ny_n(2)$

$2y_{n+2}(1+x^2) + 4x.xy_{n+1} + 2n(n-1)y_n + y_{n+1}(2x) + 2ny_n = 0$

At $x = 0$

$\Rightarrow 2y_{n+2} + 2n(n-1)y_n + 2ny_n = 0$

$\Rightarrow y_{n+2} + n^2y_n = 0$

⇒ [illegible] correct

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7. Explanation:

$y^{1/m} + y^{-1/m} = 2x$

$y^{1/m} + \frac{1}{y^{1/m}} = 2x$

$\Rightarrow \left(y^{1/m}\right)^2 + 1 = 2x\left(y^{1/m}\right)$

$\Rightarrow \left(y^{1/m}\right)^2 - 2x\left(y^{1/m}\right) + 1 = 0$

$y^{1/m} = \frac{-(-2x) \pm \sqrt{4x^2 - 4}}{2}$

$= \frac{2x \pm 2\sqrt{x^2 - 1}}{2}$

$y^{1/m} = x \pm \sqrt{x^2 - 1}$

$y = \left(x \pm \sqrt{x^2 - 1}\right)^m$

$\Rightarrow \log y = m \log\left(x \pm \sqrt{x^2 - 1}\right)$

$\frac{y'}{y} = \frac{m}{\sqrt{x^2 - 1}}$

$\Rightarrow y'\sqrt{x^2 - 1} = my$

$\Rightarrow (x^2 - 1)(y')^2 = m^2y^2$

$\Rightarrow (x^2 - 1)(2y'y'') + (2x)y'^2 = nm^2yy'$

$\Rightarrow 2(x^2 - 1)y'' + 2xy' - 2m^2y = 0$

By lebnit'z Rule

$\{(x^2 - 1)y_2\}_n = {}^nL_0y_{n+2}(x^2 - 1) + {}^nL_0y_{n+1}(2x) + {}^nL_0y_n(2)$

And $\{2xy_1\}_n = {}^nL_0y_{n+1}(2x) + {}^nL_0y_n(2)$

$(m^2y)_n = m^2y_n$

$\Rightarrow 2y_{n+2}(x^2 + 1) + (2n + 1)y_{n+1}x + n(n -$

$1)y_n(2) + 2n^2y_n + y_{n+1}2x + ny_n2 - 2m^2y_n = 0$

$= 2y_{n+1}(x^2 - 1) + (2n + 1)2y_{n+1}x + 2n^2y_n -$

$2m^2y_n = 0$

$= 1.(x^2 - 1)y_{n+2} + (2n + 1)2y_{n+1}(x) + 2n^2y_n -$

$2m^2y_m$

$(x^2 - 1)y_n + 2 + (2n + 1xy_{n+1} = m^2y_n - n^2y_n)$

$\Rightarrow (x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} = (m^2 - n^2)y_n$

$\Rightarrow$ option (1) is correct

8. Explanation:

(1) f is differentiable at $x = 0$ and f'(0) = 0

9. Explanation:

(1) $m = 0$

10. Explanation:

$u = \sin\left(\frac{x}{y}\right) x\frac{\partial u}{\partial x} + y\frac{\partial y}{\partial y}$

$\frac{\partial u}{\partial x} = \cos\left(\frac{x}{y}\right)\left(\frac{1}{y}\right).$

$\frac{\partial u}{\partial y} = \cos\left(\frac{x}{y}\right)\frac{-x}{y^2}$

$\therefore x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{x}{y}\cos\left(\frac{x}{y}\right) + y\frac{(-x)}{y^2}\cos\left(\frac{x}{y}\right)$

$= \frac{x}{y}\cos\left(\frac{x}{y}\right) - \frac{x}{y}\cos\left(\frac{x}{y}\right)$

$= 0$

$\Rightarrow$ option (1) is correct

11. Explanation:

$u(x, y)$ is Homogeous function of degree & we consider equation

$4(x, y) = x^2 + 2xy + y^2$

$\frac{\partial u}{\partial x} = 2x + 2y$

$\frac{\partial u}{\partial y} = 2x + 2y$

$\frac{\partial^2 u}{\partial r^2} = 2$

$\frac{\partial^2 u}{\partial u_2} = 2$

$\frac{\partial u}{\partial y} = 2x + 2y$

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = 2$

$\therefore x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

$= x^2(2) + 2xy(2) + y^2(2).$

$2(x^2 + 2xy + y^3)$

$= 2u$

$\Rightarrow$ option (2) is correct

12. Explanation:

(3) $\frac{\partial^2}{\partial x\, \partial y}(r^n \cos n\theta) =) n(n-1)r^{n-2}\sin(n-2)\theta$

13. Explanation:

$\theta = t^n e^{-\frac{r^2}{4t}}$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(t^n e^{\frac{-r^2}{4t}}.\frac{-2r}{4t}\right) = nt^{n-1}e^{-\frac{r^2}{4t}} +\right.$

$\left. t^n e^{-\frac{r^2}{4t}}.\frac{d}{dt}\left(\frac{-r^2}{4t}\right)\right)$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-2r^3}{4t}t^n e^{\frac{-r^2}{4t}}\right) = nt^{n-1}e^{-\frac{r^2}{4t}} + t^n e^{\frac{-r^2}{4t}}\frac{r^2}{4t^2}$

$\frac{1}{r^2}\frac{d}{dr}\left(-\frac{r^3}{2}t^{n-1}e^{\frac{-r^2}{4t}}\right) = nt^{n-1}e^{\frac{-r^2}{4t}} + t^n e^{\frac{-r^2}{4t}}.\frac{r^2}{4t^2}$

$\frac{t^{n-1}}{r^2}\left(\frac{-3r^2}{2}e^{-\frac{r^2}{4t}} + \frac{-r^3}{2}e^{\frac{-r^2}{4t}}\left(\frac{-2r}{4t}\right)\right) = nt^{n-1}e^{\frac{-r^2}{4t}} +$

$t^{n-1}e^{\frac{-r^2}{4t}}.\frac{r^2}{4t}$

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$\therefore \frac{-3}{2}t^{n-1}e^{\frac{-r^2}{4t}} + \frac{r^2}{4t}t^{n-1}e^{\frac{-r^2}{4t}}$

$= nt^{n-1}e^{\frac{-r^2}{4t}} + \frac{r^2}{4t} - e^{\frac{r^2}{4t}}t^{n-1}$

$\Rightarrow n = \frac{-3}{2}$

$\Rightarrow$ option (3) is correct

14. Explanation:

$f(x,y,z) = \log(x^3 + y^3 + z^3 - 3xyz)$

We find $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 f$

$\frac{\partial}{\partial x}f = \frac{1}{x^3+y^3+z^3-3xyz}3x^2 - 3yz$

$\frac{\partial f}{\partial y} = \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz}$

$\frac{\partial f}{\partial y} = \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz}$

$\frac{\partial f}{\partial z} = \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}$

$\left(\frac{\partial y}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial y}{\partial z}\right)x = \frac{3(x^2+y^2+z^2)-3(xy+yz+xz)}{x^3+y^3+z^2-3xyz}$

$= \frac{3}{x+y+z}$

$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\right) = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2$

$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 f = \frac{-3}{(x+y+z)^2} + \frac{-3}{(x+y+z^2)^2} + \frac{-3}{(x+y+z)^2}$

$= \frac{-9}{(x+y+z)^2}$

$\Rightarrow$ option (4) is correct

15. Explanation:

The n^{th} derivative $(x-1)^n$ w.r.t x

$f(x) = (x-1)^n$

$f'(x) = n(x-1)^{n-1}$

$f''(x) = n(n-1)\,x^{n-2}$

$f^{n-1}(x) = n(n-1)\,(n-2)\,\ldots\ldots\,(x-1)$

$f^n(x) = n(n-1)\,(n-2)\ldots 3 \times 2 \times 1$

$n!$

$\Rightarrow$ option (3) is correct

16. Explanation:

(2) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$

17. Explanation:

u is Homogeous function of x & Y of degree n.

$u(x,y) - x^n + x^{n-1}y + y^{n-1}x + y^n$

$\frac{\partial u}{\partial x} = nx^{n-1} + n-1\,x^{n-2}y + y^{n-1}$

$\frac{\partial^2 u}{\partial x^2} = n(n-1)x^{n-2} + (n-1)\,(n-2)x^{n-3}y$

$\frac{\partial u}{\partial y} = 0n\,y^{n-1} + n-1\,y^{n-2}x + x^{n-1}$

$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}(ny^{n-1} + (n-1)y^{n-2}x + x^{n-1})$

$= (n-1)y^{n-2} + (n-1)x^{n-2}$

$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = x[n(n-1)x^{n-2} + (n-1)(n-$

$2)^{n-2}xy]$

$+y[(n-1)y^{n-2} + (n-1)x^{n-2}]$

$= n(n-1)x^{n-1} + (n-1)\,(n-2)x^{n-2}y$

$+(n-1)y^{n-1} + (n-1)x^{n-2}y\,.$

$= (n-1)[nx^{n-1} + (n-2)x^{n-2}y + y^{n-1} + x^{n-2}y]$

$= (n-1)\frac{\partial i}{\partial x}$

$\Rightarrow$ option (1) is correct

18. Explanation:

$f(x) = x(x-2), a = 0, b = 1$

$f(0) = 0$

$f(1) = 1$

$f'(x) = 2x - 2$

$f(b) - f(a) = (b-a)f'(c)$

$1 - 0 = (1-0)(2c-2)$

C=1

19. Explanation:

$r = \sqrt{a^2+b^2}\ \phi = \tan^{-1}\left(\frac{b}{a}\right)$

$y = e^{ax}\cos(bx+c)$

$\frac{dy}{dx} = e^{ax}[-\sin(bx+c).b + e^{ax}\cos(bx+c)a]$

$= e^{ax}[a\cos(bx+c) - b\sin(bx+c)]$

Let

$\frac{a}{\sqrt{a^2+b^2}} = \cos x$ & $\frac{b}{\sqrt{a^2+b^2}} = \sin x$

Then $\tan\phi = \frac{b}{a}$

$\phi = \tan^{-1}\left(\frac{b}{a}\right).$

$\frac{dy}{dx} = e^{ax}\sqrt{a^2+b^2}\,[\cos\infty.\cos(bx+c) -$

$\sin\infty.\sin(bx+c)]$

$= e^{ax}(a^2+b^2)^{\frac{1}{2}}\cos(bx+c+x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^{ax}(a^2+b^2)^{\frac{1}{2}},\cos(bx+c+\infty)\right]$

$= (a^2+b^2)^{\frac{1}{2}}\frac{d}{dx}\left[e^{ax}[e^{ax}.\cos(bx+c+\phi)]\right]$

$= (a^2+b^2)^{\frac{1}{2}}\left[e^{ax}[e^{ax}.\cos(bx+c+\phi)]\right]$

$= (a^2+b^2)^{\frac{1}{2}}\left[e^{ax}\{-\sin(bx+c+\phi)\}\frac{d}{dx}(bx+c+\right.$

$\left.\phi) + \cos(bx+c+\phi)e^{ax}.\frac{d}{dx}(ax)\right]$

$= (a^2+b^2)^{\frac{1}{2}}[-e^{ax}\sin(bx+c+\phi)\times b + \cos(bx+$

$c+\phi.e^{ax}\times a\times 1)]$

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$= e^{ax}(a^2+b^2)^{\frac{1}{2}}[\cos\phi(bx+c+\phi) - b\sin(bx+c+\phi)]$

$= e^{ax}(a^2+b^2)^{\frac{1}{2}}\sqrt{a^2+b^2}$

$\left[\frac{a}{\sqrt{a^2+b^2}}\cos(bx+c+\phi) + \frac{b}{\sqrt{a^2+b^2}}\sin(bx+c+\phi)\right]$

$\frac{d^2y}{dx^2} = e^{ax}(a^2+b^2)^{\frac{3}{2}}[\cos\phi\cos(bx+c+\phi) -$

$\sin\phi\sin(bx+c+\phi)]$

$e^{ax}(a^2+b^2)^{\frac{3}{2}}(\cos(bx+c+2\phi))$

In general

$\frac{d^ny}{dx^n} = e^{ax}(a^2+b^2)^{\frac{n}{2}}(\cos(bx+c+n\phi))$

$= r^n e^{ax}(\cos(bx+c+n\phi))$

⇒ option (2) is correct

20. Explanation:

$$g(x) = \begin{cases} \frac{\sin x}{x} & if \quad x \neq 0 \\ 1 & if \quad x = 0 \end{cases}$$

$\lim_{x\to 0}\frac{\sin x}{x} = f(0) = 1$

$\therefore g(x)$ is continuous at $x = 0$

$= \lim_{x\to 0}\frac{g(x)-g(0)}{x-0} = g'(0)$

$= \lim_{x\to 0}\frac{\frac{\sin x}{x}-1}{x-0} == \lim_{x\to 0}\frac{\sin x - x}{x^2}$

$= \lim_{x\to 0}\frac{\cos x - 1}{xx}$

$= \lim_{x\to 0}\frac{-\sin x}{2}$

$= 0 = g'(0)$

$\therefore$ function is continuous as well as differentiable

⇒ option (3) is correct

21. Explanation: Given that, $f(x) = \begin{cases} \frac{x}{|x|}, x \neq \\ 0, x = 0 \end{cases} =$

$$\begin{cases} 1 : x > 0 \\ 0 : x = 0 \\ -1 : x < 0 \end{cases}$$

Consider the interval [-1, 1], then for $-1 \le x < 0$

we have $F(x) = \int_{-1}^{x} f(t)dt = \int_{-1}^{x} -1dt = -x - 1$

for $0 \le x < 1$, we have

$F(x) = \int_{-1}^{x} f(t)dt = \int_{0}^{x} f(t)dt = \int_{-1}^{0} -1dt = \int_{-1}^{x} 1dt$

$= -1 + x$

i.e. $F(x) = \begin{cases} -x-1, & -1 \le x < 0 \\ x-1, & 0 \le x \le 1 \end{cases}$

Clearly, $F'(0)$ does not exist Also $F(x)$ is continuous at $x = 0$ and F is Riemann integrable (being the continuous function over [-1,1]).But $F(x)$ is not anti-derivative of f on[-1,1].

Hence option (2) is correct.

22. Explanation:

Since $f(x) = \begin{cases} x, & 0 \le x < -1/n \\ x - \frac{(2k-1)}{2n}; & \frac{k-1}{n} < x \le \frac{k'}{n} \; k = 2,3\ldots n \end{cases}$

Clearly, f is not continuous, at $x = 1$, we have

L.H.L., $\lim_{x\to\frac{1}{n}^-} f(x) = \lim_{x\to\frac{1^-}{n}} x = \frac{1}{n}$

R.H.L., $\lim_{x\to\frac{1}{n}^+} f(x) = \lim_{x\to\frac{1^+}{n}}\left(x - \frac{3}{2n}\right)$ (Note that $k = 2$)

$= \frac{1}{n} - \frac{3}{2n} = \frac{-1}{2n}$.

Hence, $f(x)$ is not continuous at $x = \frac{1}{n}$

At, $x = \frac{r}{n}$ for any r, then

$\lim_{x\to\frac{r}{n}} f(x) = \lim_{x\to\frac{r}{n}}\left(x - \frac{2r-1}{2n}\right) = \left(\frac{r}{n} - \frac{2r-1}{2n}\right) = \frac{1}{2n}$

$= \frac{r}{n} - \frac{2r+1}{2n} = -\frac{1}{2n}$

Hence, $f(x)$ is not continuous, at each of $x = \frac{k}{n}, k =$ $1,2,\ldots n$

Hence, $f(x)$ have countable number of discontinuous

Since, A function with a countable set of discontinuities is Riemann integrable.

Hence, $f(x)$ is Riemann integrable over [0,1]

Hence option (2) is correct.

23. Explanation: Given that,

$$f(x) = \begin{cases} 0 & 0 \le x < 2 \\ 1 & 2 \le x \le 5 \\ 0 & 5 \le x \le 10 \end{cases}$$

If $0 \le x < 2$, then $F(x) = \int_0^x f(x)dx = \int_0^x 0dx = 0$

If $2 \le x \le 5$, then $F(x) = \int_0^x f(x)dx = \int_0^2 0dx +$ $\int_2^x 1dx = (x-2)$

If $5 \le x \le 10$, then $F(x) = \int_0^x f(x)dx = \int_0^2 0dx +$ $\int_2^5 1dx + \int_5^x 0dx = 3$

i.e. $F(x) = \begin{cases} 0 & 0 \le x < 2 \\ x-2 & 2 \le x \le 5 \\ 3 & 5 \le x \le 10 \end{cases}$

Clearly F(x) is continuous for all x.

But $F(x)$ is not differentiable at $x = 2$ and $x = 5$, as both Left and Right hand derivative at x=2 and x-5 are not same.

Hence, option (3) is correct.

24. Explanation:

Given function is

$f(x) = \begin{cases} \sin x, & x \ge 0 \\ 1 - \cos x, & x < 0 \end{cases}$

Now, let us check differentiability

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$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$

Now for, $x \geq 0$

$\Rightarrow \lim_{x\to 0}\frac{\sin x-0}{x-0}=1$

And for $x<0$

$\Rightarrow \lim_{x\to 0}\frac{1-\cos x-1+1}{x}=DNE$

$\Rightarrow$ The function f is not differentiable

Now, let us check continuity

$f(0^+)=\lim_{h\to 0^+} f(x)=\lim_{h\to 0^+}\sin h=0$

And

$f(0^-)=\lim_{h\to 0^-} f(x)=\lim_{h\to 0^-} 1-\cos h=1-1=0$

$\Rightarrow$ The function is continuous

$\Rightarrow$ option (2) is correct

25. Explanation:

Given $F:(-1,1)\to\mathbb{R}$ is a differentiable function

$\Rightarrow F'(x)=2x$

$\Rightarrow F'(0)=0$

Now, if $\delta=0.1$

$\Rightarrow x=\frac{\delta}{2}=0.05$

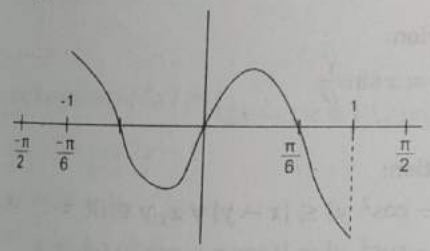
$\Rightarrow F(x)>0$, for $(-\delta,\delta)$ for $\delta=0.1$

$\Rightarrow$ option (c) is incorrect

a) Let $F(x)=\sin 6x$

$F'(x)=6\cos 6x$

$F'(0)=6>0$



But $F(x)<F(0)$ as $x\in(0.52,1)$

$\Rightarrow$ option (a) is incorrect

$\Rightarrow$ option (2) is correct

26. Explanation:

$t=\frac{\pi}{4}, \omega=xy^2z\ x=\cos t, y=\sin t\ z=t$

$\frac{\partial\omega}{\partial x}=y^2z=t\sin^2 t=\frac{\pi}{4}\left(\frac{1}{\sqrt{2}}\right)^2=\frac{\pi}{8}$

$\frac{\partial\omega}{\partial y}=2xyz=2t\cos t\sin t=t\sin 2t=\frac{\pi}{4}\sin\left(\frac{t}{4}\right)=\frac{\pi}{4}$

$\frac{\partial\omega}{\partial z}=xy^2=\cos t\sin^2 t=\sin^2\left(\frac{t}{2}\right)\cos\left(\frac{t}{4}\right)=\frac{1}{2\sqrt{2}}$

$\frac{\partial x}{\partial t}=-\sin t=\frac{-1}{\sqrt{2}}\qquad \frac{\partial y}{\partial t}=\cos t=\frac{1}{\sqrt{2}}\frac{\partial e}{\partial t}=1$

$\frac{\partial\omega}{\partial z}=\frac{\partial\omega}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial\omega}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial\omega}{\partial z}\frac{\partial z}{\partial t}$

$=\frac{\pi}{8}\left(\frac{1}{\sqrt{2}}\right)+\frac{\pi}{4}\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2\sqrt{2}}$

$=\frac{-\pi}{8\sqrt{2}}+\frac{\pi}{4\sqrt{2}}+\frac{1}{2\sqrt{2}}$

$=\frac{\pi+4}{2\sqrt{2}}$

27. Explanation:

$n+2>1$

$n>-1$

28. Explanation:

$f(x)=x^2\cos(x^2)$

$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$

$=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}\cdots$

$x^2\cos(x^2)=x^2\left[1-\frac{x^4}{2!}+\frac{x^8}{4!}-\frac{x^{12}}{6!}+\cdots\right]$

$=x^2-\frac{x^6}{2!}+\frac{x^{10}}{4!}-\frac{x^{14}}{6!}+\cdots$

Coefficient of x^8 is zero

29. Explanation:

$g:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that

$|g'(x)\leq|x$

$f(x)=x+\epsilon\, g(x)$

$f'(x)=1+\epsilon\, g'(x)$

$f'(x)=1+\epsilon\, g'(x)$

30. Explanation:

$f(x)=\int_x^{x^2} e^{b^2}dt\ x>1$

$f(x)=\int_x^{x^2} 2te^{t^2}dt+e^{x^4}(2x)-e^{x^2}$

(by using leigniz rule)

$\int_x^{x^2} 2te^{t^2}dt$

Put $t^2=y$

$2tdt=dy$

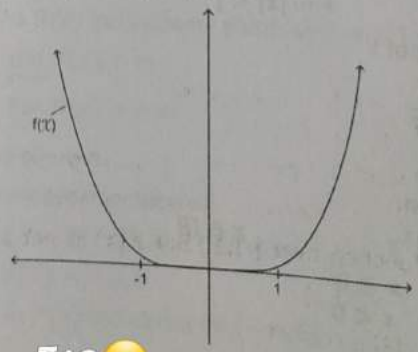
$t=x\ y=x^2$

$t=x^2\ y=x^4$

$\int_x^{x^2} 2te^{t^2}dt=\int_{x^2}^{x^4} e^y dy=[e^y]_{x^2}^{x^4}=e^{x^4}-e^{x^2}$

$f'^{(x)}=e^{x^4}-e^{x^2}+e^{x^4}(2x)-e^{x^2}$

$=e^{x^4}(1+2x)-2e^{x^2}$



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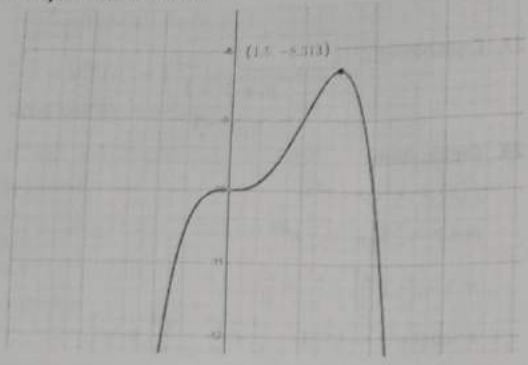
$f(x)$ increasing function

$\Rightarrow$ both statements are false

31. Explanation: -8.313



32. Explanation:

$x > 0$

$$F(x) = \int_0^x \frac{\sin xy}{y} dy$$

$$\frac{d}{dx}(F(x)) = \frac{d}{dx}\int_0^x \frac{\sin xy}{y} dy$$

$$F'(x) = \int_0^x \frac{\sin xy}{y} dy + \frac{\sin x^2}{x}$$

$$F'(x) = \int_0^x \frac{\sin xy}{y} dy + \frac{\sin x^2}{x}$$

$$= \left[\frac{\sin xy}{x}\right]_0^x + \frac{\sin x^2}{x}$$

$$F'(x) = \frac{2\sin x^2}{x}$$

33. Explanation:

(1) $f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$

(2) $f(x) = |x|x.$

34. Explanation: Maclaumin series expansion

$$F(x) = \sin^{-1}(2) = x + \frac{x^3}{2.3} + \frac{1.3x^5}{2.4.5} + \frac{1.3.5x^7}{2.4.6.7} + \cdots |x| < |$$

Coefficient of x^7

$$\frac{1.3.5}{2.4.6.7}$$

35. Explanation:

(1) $F(x) = |x|^3 x \qquad x \in IR$

$$= \begin{cases} x^4 & x \geq 0 \\ -x^4 & x < 0 \end{cases}$$

$$\lim_{n\to 0^+} F(x) = \lim_{n\to 0^+} \frac{F(0+n) - F(0)}{n}$$

$$= \lim_{n\to 0^+} \frac{F(n) - F(0)}{n} = \lim_{n\to 0} \frac{n^4}{n} = 0$$

$$\lim_{n\to 0^-} F(x) = \lim_{n\to 0^-} \frac{F(0-n) - F(0)}{n} = \lim_{n\to 0^-} \frac{n^4}{n} = 0$$

Clearly $F(x)$ is differentiable at 0

$\Rightarrow F(x)$ is differentiable everywhere

(2) $|F(x) - F(y)| \leq |x - y|^{\sqrt{2}}$

$\delta = \sqrt{2} >$

$F(x)$ is correct

$\Rightarrow F$ is differentiable everywhere

(3) $f(x) = \begin{cases} x^3 \sin \frac{1}{\sqrt{|2|}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{n\to 0^+} F'(0) = \lim_{n\to 0^+} \frac{F'(h) - F(0)}{h} = \lim_{n\to 0} \frac{n^2 \sin\left(\frac{1}{\sqrt{|n|}}\right)}{n} = 0$$

$$\lim_{n\to 0^+} F'(0) = \lim_{n\to 0^+} \frac{F'(h) - F(0)}{0} = \lim_{n\to 0} \frac{n^2 \sin\left(\frac{1}{\sqrt{|n|}}\right)}{n} = 0$$

$f(x)$ is differentiable at 0

$\Rightarrow F(x)$ is differentiable everywhere

36. Explanation:

$$\log\left(x + \sqrt{1 + x^2}\right) = x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^1 + \ldots$$

$$\Rightarrow \frac{-5}{112}$$

38. Explanation:

(1) $f(x) = x \sin \frac{1}{\sqrt{x}}$

39. Explanation:

(1) $|\cos^2 x - \cos^2 y| \leq |x - y| \; \forall \; x_1 y \in IR$

$|\cos^2 x - \cos^2 y| = |(\cos x - \cos y)(\cos x + \cos y)| < |x - y||\cos x + \cos y|$

$|\cos^2 x - \cos^2 y| \leq |x - y|$

(2) $|f(x) - f(y)| \leq |x - y|^{\delta}$

$\delta >$, then f is constant

(c) $F, IR \to IR$ be continuously differentiable

$|f'(x)| \leq \frac{4}{5}$

$\Rightarrow \exists$ unique function

40. Explanation:

1. $\lim_{x\to a} \frac{a^n f(x) - x^n f(a)}{x - a}$

$\frac{0}{0}$ form

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$\lim_{x\to a} \frac{a^n f'(x) - f(a)x^n \log x}{1}$

$a^n f'(a) - f(a)a^n \log a$

$a^n(f'(a) - f(a)a^n \log a)$

41. Explanation:

$f(x) = x^2e^{3x}$

$f'(x) = 2xe^{3x} + 3x^2e^{3x} = e^{3x}(2x + 3x^2)$

$f''(x) = 3e^{3x}(2x + 3x^2) + e^{3x}(2x + 3x^2) =$

$e^{3x}(6x + 9x^2 + 2 + 6x) = e^{3x}(9x^2 + 12x + 2)$

$f''(x) = 3e^{3x}(9x^2 + 12x + 2) + e^{3x}(18x + 12)$

$= e^{3x}(27x^2 + 36x + 6 + 18x + 12)$

$= e^{3x}(27x^2 + 54x + 18)$

$f^n(x) = e^{3x}(3^n x^2 + 63^{n-1}nx + 3^{n-2}n(n-1))$

$f^n(x) = 3^{n-2}e^{3x}[9x^2 + 6nx + n(n-1)]$

42. Explanation:

$F : (0,\infty) \to (0,\infty)$

$\lim_{x\to 0}\left(\frac{F(x)}{F(a)}\right)^{\frac{1}{\log x - \log a}}$

1^∞ form

$e^{\lim_{x\to a}\frac{1}{\log x - \log a}\left[\frac{F(x)}{F(a)}-1\right]}$

$e^{\lim_{x\to a}\frac{F(x)-F(a)}{F(a)(\log x - \log a)}}$ $\left(\frac{0}{0}\right)$ form

$e^{\lim_{x\to a}\frac{F(x)}{F(a)(1/x)}}$

$= e^{\frac{aF(a)}{F(a)}}$

43. Explanation: $F(x) = \begin{cases} x^2 & x \in (0\ 2) nQ \\ 2x - 1 & x \in (0,2) nQ^c \end{cases}$

$\lim_{x\to a} x^2 = \lim_{x\to a} 2x - 1$

$a^2 = 2a - 1$

$a^2 - 2a + 1 = 0$

$\Rightarrow a = 1$

For differentiable only x=1

44. Explanation:

$$log\left(\sqrt{1+x^2}\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^{2n}$$

45. Explanation:

$F : (0,\infty) \to (0,\infty)$

$\lim_{x\to 0}\left(\frac{F(x)}{F(a)}\right)^{\frac{1}{\log x - \log a}}$

1^∞ form

$e^{\lim_{x\to a}\frac{1}{\log x - \log a}\left[\frac{F(x)}{F(a)}-1\right]}$

$e^{\lim_{x\to a}\frac{F(x)-F(a)}{F(a)(\log x - \log a)}}$ $\left(\frac{0}{0}\right)$ form

$e^{\lim_{x\to a}\frac{F(x)}{F(a)(1/x)}}$

$= e^{\frac{aF(a)}{F(a)}}$

46. Explanation:

(1) $F(x) = \begin{cases} \tan^{-1}\left(\frac{1}{|x|}\right) & x \neq 0 \\ \frac{\pi}{2} & if\ x = 0 \end{cases}$

$F(x) = \begin{cases} \frac{1}{x^2+1} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$\lim_{x\to 0} F'(0) \neq F'(0)$

$\Rightarrow F(x)$ is not differential

(2) $F(x) = |x|^{\frac{1}{2}}x$

$F(x) = \begin{cases} x^{3/2} & x \geq 0 \\ -x^{3/2} & x < 0 \end{cases}$

$\lim_{x\to 0^+} F(x) = \lim_{x\to 0^+} \frac{x^{3/2}}{2} = 0$

$\lim_{x\to 0^-} F(x) = \lim_{x\to 0^-} \frac{x^{3/2}}{2} = 0$

$F(x) = \begin{cases} \frac{3}{4}x^{1/2} \\ -\frac{3}{4}(x)^{1/2} \end{cases}$

$\lim_{x\to 0^+} F'(x)\ F'(0)$

$\Rightarrow F(x)$ is differentiable

(3) $F(x) = \begin{cases} x^2\left|\cos\frac{\pi}{2}\right| & x \neq 0 \\ 0 & x = 0 \end{cases}$

$F(x)$ is continuous at $F(x)$

47. Explanation: $F(x) = 2x^3 - 9x^2 + 12x + 4$

$F'(x) = 6x^2 - 18x + 12$

$F'(x) = 0$

$\Rightarrow 6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0$

$x = 2, 1$

$\Rightarrow F(x)$ has extreme points at $x = 2, 1$

$\lim_{x\to\infty} F(x) = \infty$

$\lim_{x\to\infty} F(x) = -\infty$

$F(0) = 4$

We consider interval

$(-\infty, 2)$ $(1,2)$ $(2,\infty)$

$F(0) = 4 > 0$

$\Rightarrow F(x)$ creasing on $(-\infty, 2)$

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$F^{11}(x) = 12x - 18$

$F^{11}(1) = -6 < 0$

$F^{11}(2) = 6 > 0$

$\Rightarrow x = 1$ is maximum point + $x = 2$ is minimum

$\Rightarrow F(x)$ decreasing in $-1 < x < 2$

48. Explanation:

(1) $f(x) = x|x| = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$

$f'(x) = \begin{cases} -2x & x < 0 \\ 2x & x \geq 0 \end{cases}$

$\lim_{x \to 0^-} f'(x) = 0$

$\lim_{x \to 0^+} f'(x) = 0$

$\lim_{x \to 0} f'(x) = 0 = f'(0)$

$\Rightarrow f(x)$ is differentiable

(2) $f(x) = [x] \sin^2 \pi x$

$f'(a) = \lim_{n \to 0} \frac{f(a+n) - f(a)}{k} a \in \mathbb{R}$

$f'(a) = \lim_{n \to 0} \frac{[a+h] \sin^2 \pi(ah) - [a] \sin^2 \pi a}{h}$

$\frac{0}{0}$ form

$\lim_{x \to 0} f'(x) = 0 = f'(0)$

$\Rightarrow f(x)$ is differentiable

(3) $f(x) = |x - a|$ P is differentian at $x = 0$ if $P > 1$

Here $P = \frac{1}{2}$

$\Rightarrow f(x)$ is not differentiable

49. Explanation:

(1) Let f' is continuously differentiable in $[a, b]4$

If $f(a) = f(b)$

By Roll's theorem & $c \in (a, b)$ $f'(c) = 0$

As f is twice differentiable

$\Rightarrow f'$ is differentiable on (a,b) & $f'(a) = f'(c) = 0$

By Roll's theorem

$I\ x_o \in (a, b)$ such that $f''(x_0) = 0$

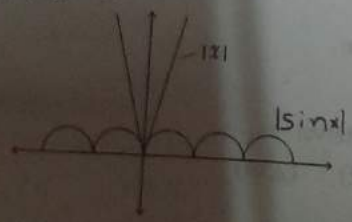
So statement (a) true

(2) f is continuously differentiable in $[a, b]$

$f(a) = f(b)$ & $f'(a) = f'(b)$

By rolls' theorem $x \in (a, b)$

$f'(x_1) = 0$

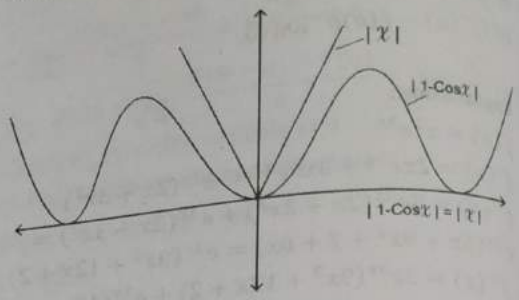


1.

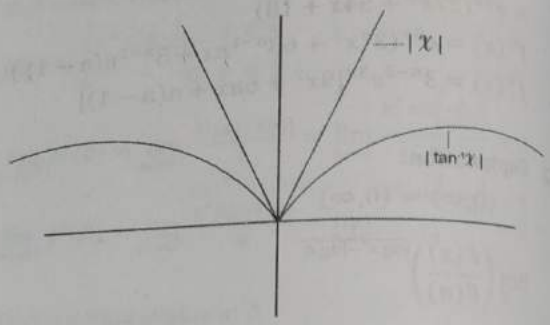
So $|\sin x| \leq |x|$

So statement a correct

2.



3.



$|\tan^1 x| \leq |x| \qquad \forall x \in \mathbb{R}$

50. Explanation: $F(x) = |x|^{\frac{5}{2}}$

Since $f(x) = |x - a|^{\alpha}$ is different at a for $\alpha > 1$

$\Rightarrow F(x)$ is differentiable at 0

$F(x) = \begin{cases} x^{5/2} & x \geq 0 \\ (-x)^{5/2} & x < 0 \end{cases}$

$F'(0^+) = \lim_{n \to 0^+} \frac{F(0+n) - F(0)}{n}$

$= \lim_{n \to 0^+} \frac{n^{5/2}}{n} = 0$

$F'(0^-) = \lim_{n \to 0^-} \frac{F(a-n) - F(0)}{n}$

$= \lim_{n \to \infty} \frac{(-n)^{5/2}}{n} = 0$

$F'(x) = \begin{cases} \frac{5}{2} x^{3/2} & x \geq 0 \\ \frac{5}{2}(-x)^{3/2} & x < 0 \end{cases}$

$F'(x) = \frac{5}{2} |x|_x^{\frac{1}{2}}$

option (3) correct

51. Explanation:

$F(x) = x^3$

$\lim_{x \to \infty} x^3 \Rightarrow \infty$

$\lim_{x \to \infty} x^3 \to -\infty$

$\therefore$ by continuous extension theorem $F(x)$ is not [illegible] continue

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52. Explanation:

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x} dx$ ...1

$\int_a^b F(x)dx = \int_a^b F(a+b-x)dx$

$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x+\sin x} dx$ ...2

From 1 to 2

$2I = \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x+\cos e}$

$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}\,dx}{\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1}$

$\tan\frac{x}{2} = t$

$\frac{1}{2}\sec^2\frac{x}{2}dx\,dt$

$2I = \int \frac{2dt}{t^2+2t+1}$

$I = \int_0^1 \frac{dt}{-(t^2+2t+1)}$

$= \int_0^1 \frac{1}{(t-1)^2-(\sqrt{2})^2} dt$

$\int \frac{1}{x^2-a} dt = \frac{1}{2a}\log\left[\frac{a-a}{x+a}\right]$

$I = \frac{1}{2\sqrt{2}}\log\left[\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right]_0^1$

$I = \frac{1}{2\sqrt{2}}\left(0 - \log\left|\frac{-1-\sqrt{2}}{-1+\sqrt{2}}\right|\right)^0$

$= \frac{1}{2\sqrt{2}}\left(\log(\sqrt{2}+1) + \log(\sqrt{2}+1)\right)$

$= \frac{\log(\sqrt{2}+1)}{\sqrt{2}}$

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Unit 1.8: Sequence & Series of Function

[PRACTICE QUESTIONS]

1. Let $\{f_n\}$ be a sequence of functions defined on [0, 1]. Determine $f(x) = \lim_{n\to\infty} f_n(x)$, and determine if the following claim is true or false:

$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

$f_n(x) = n^2x(1-x^2)^n$.

(1) True (2) False

(3) Cannot decided (4) None of the above

2. Let $f_n(x) = n^2x(1-x)^n$. Evaluate:

$\lim_{n\to\infty} f_n(x)$

for $0 < x < 1$.

(1) $-\infty$ (2) ∞

(3) $\propto$ (4) none of the above

3. Which of the following functions is not continuous at every point of their respective domains of definition?

(1) $f(x) = \lim_{n\to\infty} \frac{x^2e^{nx}+x}{e^{nx}+1}, x \in \mathbb{R}$.

(2) $f(x) = \lim_{n\to\infty} \frac{x^2e^{nx}+\cos x}{e^{nx}+1}, x \in \mathbb{R}$.

(3) $f(x) = \lim_{n=>\infty} \frac{1}{n}\log(e^n + x^n), x \geq 0$.

(4) None of the above

4. Which of the following functions are uniformly continuous over their respective domains of definition?

(1) $f(x) = \sum_{n=1}^{\infty} \frac{g(x-n)}{2^n}, x \in \mathbb{R}$,

where $g : \mathbb{R} \to \mathbb{R}$ is a bounded and uniformly continuous function.

(2) $f(x) = \cos x \cos\frac{\pi}{x}, x \in]0,1[$.

(3) $f(x) = \sin x \sin\frac{\pi}{x}, x \in]0,1[$.

(4) None of the above

5. For the function $f(x)$ on the real line $\mathbb{R}$ defined below, which of the following statements about f is true?

Choose all the correct options: $f(x) := \sum_{n\geq 1} \frac{\sin(x/n)}{n}$

(1) f is continuous but not uniformly continuous on $\mathbb{R}$.

(2) f is uniformly continuous on $\mathbb{R}$.

(3) f is differentiable on $\mathbb{R}$.

(4) f is an increasing function on $\mathbb{R}$.

6. For n a positive integer, let $f_n(x)$ be the continuous function $1/(1+nx)$ with domain the positive real numbers. Let $f(x)$ be the pointwise limit of the sequence $\{f_n(x)\}_{n\geq 1}$ of functions. On which of the following intervals is the convergence $f_n \to f$ not uniform? Choose all the correct options:

(1) (0,1) (2) (1,2)

(3) $(2, \infty)$ (4) None of the above.

Answer Key					
1	2	3	4	5	6
1	2	4	1	3	1

EXPLANATIONS

[PRACTICE QUESTIONS]

1. Explanation:

1. $f_n(x) = n^2x(1-x^2)^n \qquad x \in [0\ 1]$

$\lim_{n\to\infty} f_n(0) = 0$

$\lim_{n\to\infty} f_n(1) = 0$

Let $0 < x < 1$

$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} n^2x(1-x^2)^n = 0$

$\Rightarrow \lim_{n\to\infty} f_n(x) = 0$

$\Rightarrow f_n(x)$ converges point wise

2. $\lim_{n\to\infty} \int_0^1 fn(x)dx = \lim_{n\to\infty} \int_0^1 n^2x(1-x^2)^n dx$

$= \lim_{n\to\infty} n^2 \int_0^1 x(1-x^2)^n dx$

Put $x - x^2 = t$

$-2xdx = dt$

$= \lim_{n\to\infty} n^2 \int_0^1 t^n \left(\frac{-1}{2}\right) dt$

$= \lim_{n\to\infty} \frac{-n^2}{2}\left[\frac{t^{n+1}}{n+1}\right]_0^1$

$= \lim_{n\to\infty} \int_0^1 fn(x)dx = \lim_{n\to\infty} \frac{-n^2}{2(n+1)}$ DNE

$\int_0^1 f(x) = 0$

$\Rightarrow \lim_{n\to\infty} \int_0^1 fn(x)dx \neq \int_0^1 f(x)$

2. Explanation:

$F_n(x) = n^2x(1-x)^n$

$\lim_{n\to\infty} Fn(x) = \lim_{n\to\infty} n^2x(1-x)^n$

For $0 < x < 1$

$0 > -x > -1$

$1 > 1-x > 0$

$0 < 1-x < 1$

As $n \to \infty$ $(1-x)^n \to 0$

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$\lim_{n\to\infty} Fn(x) \to 0 \text{ as } n \to \infty$

3. Explanation:

1. $F(x) = \lim_{n\to\infty} \frac{x^2 e^{nx}+x}{e^{nx}+1}$

$= \lim_{n\to\infty} \frac{x^2 + \frac{x}{e^{nx}}}{1+\frac{1}{e^{nx}}}$

$F(x) = x^2$

$\Rightarrow F(x)$ is continuous

2. $\lim_{x\to\infty} \frac{x^2 e^{nx}+\cos x}{e^{nx}+1}$ $\qquad x \in \mathbb{R}$

$\lim_{x\to\infty} \frac{x^2 + e^{nx}\cos x}{1+e^{-nx}}$

$$F(x) = \begin{cases} \cos x & x < 0 \\ \frac{1}{2} & x = 0 \\ x^2 & x > 0 \end{cases}$$

$F(x)$ is not continuous

3. $F(x) = \lim_{n\to\infty} \log\left(x^n\left(1+\frac{e^n}{z^n}\right)\right)^{\frac{1}{n}}$

$= \lim_{n\to\infty} \left(\log x + \log(1+\left(\frac{e}{x}\right)^n\right)^{\frac{1}{n}}$

$$(x) = \begin{cases} 1 & 0 \le x < e \\ 1 & x = e \\ \log x & x > e \end{cases}$$

$\Rightarrow F(x)$ is continuous

4. Explanation:

1. $F(x) = \sum_{n=1}^{\infty} \frac{=g(x-n)}{2^n}$ $\qquad x \in \mathbb{R}$

As $g(x)$ is uniformly continuous then linear combination of $g(x)$ is also uniformly continuous

$\Rightarrow F(x)$ is U.C

2. $F(x) = \cos x \cos\left(\frac{\pi}{x}\right)$ $\qquad 2 \in (0,1)$

$\lim_{x\to 1} \cos x \cos\left(\frac{\pi}{x}\right) = -\cos(1)$

$\lim_{x\to 0} \cos x \cos\left(\frac{\pi}{x}\right) = -DNE$

$F(x)$ is not uniformly continuous

3. $F(x) = \sin x \sin\frac{\pi}{x}$

5. Explanation:

$\sum_{n\ge 1} \frac{\sin(x/n)}{n}$

$\left|\frac{\sin(x/n)}{n}\right| \le \frac{\frac{x}{n}}{n} = \frac{x}{n^2}$

$\Rightarrow \sum_{n\ge 1} \frac{\sin(x/n)}{n}$ converges uniformly

$\Rightarrow F(x)$ is continuous over IR

$F'(x) = \sum_{n=1}^{\infty} \frac{\cos(x/n)}{n}$

$\left|\frac{\cos(x/n)}{n}\right| \le \frac{1}{n^2}$

$\Rightarrow \sum \frac{\cos(x/n)}{n^2}$ is coverges uniformly

$\Rightarrow F'(x)$ is continuous

$\Rightarrow F(x)$ is differentiable over IR

6. Explanation: $Fn(x) = \frac{1}{1+nx}$

1. (0,1)

$x \in (0,1)$

$\lim_{n\to\infty} Fn(x) = \lim_{n\to\infty} \frac{1}{1+nx} = 0$

$\lim_{n\to\infty} Fn(x) = 0 = F(x)$

$Mn = sum|Fn(x) - F(x)|$

$= \sup_{x \in (0,1)} \left|\frac{1}{1+nx}\right|$

$\phi_n(x) = \frac{1}{1+nx}$

Let $x = \frac{1}{n} \to 0$ as $n \to \infty$

$Mn = \frac{1}{2} \neq 0$

$\Rightarrow <Fn(x)>$ not converges uniformly.

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Unit 1.9: Function of Several Variables

[CUET-PG]

[CUET PG 2022]

1. If $u(x,y) = \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right), 0 < x < y < 1.$
Then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$
(1) $-\frac{1}{2}\tan\tan u$ (2) $-\frac{1}{2}\cot\cot u$
(3) $\sin\sin u$ (4) $\cos\cos u$

[CUET PG 2022]

2. Let $f : R^2 \to R$ be defined by $f(x,y) = \sqrt{|xy|}$, then
(1) f_x, f_y do not exist at (0, 0)
(2) directional derivative of f exists at (0, 0) in every non-zero direction $u = (u_1, u_2)$ in R^2
(3) f is differentiable
(4) f is not differentiable

[CUET PG 2022]

3. Let $f : R \to R$ be defined by
$f(x,y) = \{\frac{x^2-y^2}{x^2+y^2}\ if\ x^2+y^2 \neq 0\ 0\ if\ x = 0 = y$
Let $f(x,y) = a$ and $f(x,y) = b$
Then
(1) $a = b = 1$ (2) $a = b = -1$
(3) $a = 1$ and $b = -1$ (4) $a = -1$ and $b = 1$

[CUET PG 2022]

4. If $z = f(x,y)$ is a homogeneous function of x, y of degree n, then
A. $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$
B. $z = x^n \emptyset\left(\frac{y}{x}\right)$
C. $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$
D. $\frac{\partial z}{\partial x} + y\frac{\partial^2 z}{\partial x \partial y} = n\frac{\partial z}{\partial y}$
E. $\frac{\partial z}{\partial x} + x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y} = n\frac{\partial z}{\partial x}$
Choose the correct answer from the options given below:
(1) A, B, D only (2) A, C, D only
(3) B, C, E only (4) A, B, E only

[CUET PG 2023]

5. If $f: R^2 \to R^2$ is a function defined as $f(x,y) = \{\frac{x}{\sqrt{x^2+y^2}}, x \neq, y \neq 0\ 2, x = 0, y = 0$ then, which of the following is correct?
(1) $f(x,y)$ is continuous at origin.
(2) $f(x,y)$ is differentiable at origin.
(3) $f(x,y)$ exists and is equal to 2.
(4) $f(x,y)$ is not continuous at origin.

[CUET PG 2023]

6. If $u = \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is
(1) $\frac{1}{2}\cot u$ (2) $-\frac{1}{2}\cot\cot u$
(3) $-\frac{1}{2}\tan\tan u$ (4) $\frac{1}{2}\tan\tan u$

Answer Key					
1	2	3	4	5	6
2	4	3	3	1	2

[CU-CET]

[CUCET 2016]

1. Consider the function $f : R \to R$ of two variables defined by
$f(x,y) = \{0, if\ (x,y) = (0,0)\ \frac{xy^2}{x^2+y^2}, if\ (x,y) \neq (0,0)$.
Determine which one of the following facts about f is not true.
(1) f is differentiable at (0,0)
(2) f is continuous at (0,0)
(3) f has directional derivative at (0,0) in the direction of any vector $u = (a,b) \neq (0,0)$
(4) partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ at $(0,0)$exist

[CUCET 2016]

2. The function $f : R^2 \to R$ given by $f(x,y) = xy$
(1) has a critical point at (0,0) which is a relative minimum
(2) has a critical point at (0,0) which is a relative maximum
(3) has a critical point at (0,0) which is a saddle point
(4) has no critical point

[CUCET 2017]

3. Suppose $\alpha = \frac{sinsin\,(x^2+y^2)}{x^2+y^2}$; $\beta = \frac{x^2-y^2}{x^2+y^2}$ then which of the following statements is true?
(1) α exists but β does not exist
(2) α does not exist but β exists
(3) α, β do not exist
(4) both α, β exist

[CUCET 2017]

4. For the function $f: R^2 \to R$ defined by $f(x,y) = x^3 + y^3 - \ldots - 12y + 20$, which of the following

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(1) f has maximum at (1, 2)
(2) f has minimum at (-1,-2)
(3) f has maximum at (1, 2) and minimum at (-1, -2)
(4) The saddle points of f are (-1, 2) and (1, –2)

[CUCET 2017]

5. Consider the function $f: R^2 \to R$ of two variables defined by
$f(x,y) = \frac{xy}{x^2+y^2}, x^2 + y^{2'} \neq 0$
$0, x^2 + y^2 = 0$
Determine which one of the following facts about f is true.
(1) f is continuous at (0, 0)
(2) f has removable discontinuity at (0, 0)
(3) f is not differentiable at (0, 0)
(4) none of the above

[CUCET 2017]

6. If $f: R^2 \to R$ be defined by $f(x,y) = x^2 + y^2$ if x and y are rational
0 otherwise
then which of the following statements is true?
(1) f is not continuous at (0, 0)
(2) f is continuous at (0, 0) but not differentiable at (0, 0)
(3) f is differentiable only at (0, 0)
(4) f is differentiable everywhere

[CUCET 2017]

7. Consider the function $f: R^2 \to R$ defined by $f(x,y) = x^y$. Determine which of the following is true for f.
(1) $f_x(a,0) = 3$ for any constant a
(2) $f_y(e,0) = 0$
(3) $f_{xy}(1,0) = 0$
(4) $f_{yx}(1,1) = 1$

[CUCET 2019]

8. If $f(x,y)$ is a homogeneous function of degree n in x and y and has continuous partial derivatives, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial x}$ is equal to
(1) f
(2) nf
(3) 0
(4) $n(n-1)f$

[CUCET 2019]

9. $(x^2 + 2x - y^2)$ equals
(1) 0
(2) -7
(3) 7
(4) -1

[CUCET 2021]

10. Let $(x,y) \in R^2$. Which one of the following is true?
(1) $\frac{2xy^2}{x^2+y^4}$ exists
(2) $\frac{2xy^2}{x^2+y^4}$ does not exists
(3) $\frac{2xy^2}{x^2+y^4}$ is continuous at (0,0,)
(4) None of these

[CUCET 2021]

11. Let $(x,y) \in R^2$. Consider the function $f: R^2 \to R$ such that $f(x,y) = x^2 + 2xy + y^2$. Then
(1) The function f is a non-homogeneous function
(2) The function f is a homogeneous function of degree 1
(3) The function f is a homogeneous function of degree 2
(4) The function f is a homogeneous function of degree 0

Answer Key									
1	2	3	4	5	6	7	8	9	10
1	3	1	4	3	2	4	2	3	2
11									
3									

[PRACTICE QUESTIONS]

1. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, then the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ is
(1) $\sin\theta$
(2) $r\sin\theta$
(3) $r^2\sin\theta$
(4) $\sin\theta\sin\phi$

2. If $x = r\cos\theta, y = r\sin\theta, 0 \leq 2\pi, r > 0$, then $\frac{\partial(x,y)}{\partial(r,\theta)}, \frac{\partial(r,\theta)}{\partial(x,y)}$ is
(1) 0
(2) 1
(3) -1
(4) 2

3. If $V = (x^2 + y^2 + z^2)^{-1/2}$, then $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ is equal to
(1) O
(2) V
(3) $2V$
(4) $3V$

4. Let the function $F: \mathbb{R}^2 \to \mathbb{R}$ be defined by f(0,0) and $f(x,y) = \frac{x^3-y^3}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Then
(1) f is continuous on $\mathbb{R}^2$
(2) f is continuous at all points o $\mathbb{R}^2$ except at (0,0)
(3) $f_x(0,0) = f_y(0,0)$
(4) [illegible]d

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5. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Let $D_u f(0,0)$ denote the directional derivative of f at (0, 0) in the direction $u = (u_1, u_2) \neq (0,0)$. Then f is

(1) continuous at (0, 0) and $D_u f(0,0)$ exist for all u

(2) continuous at (0, 0) but $D_u f(0,0)$ does not exist for some $u \neq (0,0)$

(3) not continuous at (0, 0) but $D_u f(0,0)$ exist for all u

(4) not continuous at (0, 0) and $D_u f(0,0)$ does not exist for some $u \neq (0,0)$

6. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \frac{x^2-y^2}{1+x^2+y^2}$$

Then

(1) $\frac{\partial^2 f}{\partial x \partial y}(0,0)$ and $\frac{\partial^2 f}{\partial y \partial x}(0,0)$ exist but are not equal

(2) $\frac{\partial^2 f}{\partial x \partial y}(0,0)$ exist but $\frac{\partial^2 f}{\partial y \partial x}(0,0)$ does not exist

(3) $\frac{\partial^2 f}{\partial y \partial x}(0,0)$ exist but $\frac{\partial^2 f}{\partial x \partial y}(0,0)$ does not exist

(4) $\frac{\partial^2 f}{\partial x \partial y}(0,0)$ and $\frac{\partial^2 f}{\partial y \partial x}(0,0)$ exist are equal

7. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \sqrt{|xy|}$. Then at orgin

(1) f is continuous and $\frac{\partial f}{\partial x}$ exists

(2) f is discontinuous and $\frac{\partial f}{\partial x}$ exists

(3) f is continuous and $\frac{\partial f}{\partial x}$ dos not exists

(4) f is discontinuous but $\frac{\partial f}{\partial x}$ exists

8. Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y}; & x^2 \neq -y \\ 0; & x^2 = -y \end{cases}$$

Then

(1) directional derivative does exist at (0,0)

(2) f is continuous at (0, 0)

(3) f is differentiable at (0, 0)

(4) each directional derivative exists at (0, 0) but is not continuous

9. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}; & (x,y) \neq (0,0) \\ 0; & (x,y) = (0,0) \end{cases}$$

Then

(1) f is not continuous at (0, 0) but all directional derivatives of f at (0,0) exist

(2) f is continuous in $\mathbb{R}^2$ and all directional derivatives of f at (0,0) exist

(3) f is continuous in $\mathbb{R}^2$ but not all directional derivatives (0,0) exist

(4) f is set continuous at (0,0) and not all directional derivatives at (0, 0) exist

10. The rate of change of $f(x,y) = 4y - x^2$ at the point (1,5) in the direction from (1, 5) to the point (4,3) is

(1) $\frac{-6}{\sqrt{5}}$ (2) $\frac{-14}{\sqrt{13}}$

(3) $\frac{-12}{\sqrt{5}}$ (4) $\frac{-19}{\sqrt{13}}$

11. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = |x| + |y|$. Then which one of the following statements is?

(1) f is continuous at (0, 0) and $f_x(0,0) \neq f_y(0,0)$.

(2) f is continuous at (0, 0) and $f_x(0,0) = f_y(0,0)$.

(3) f is discontinuous at (0, 0) and $f_x(0,0) \neq f_y(0,0)$.

(4) f is continuous at (0, 0) and f_x and f_y does not exist not exist at (0, 0)

12. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a map that sends $(x,y) \in \mathbb{R}^2$ to $f(x,y)$. Which one of the following is necessarily true?

(1) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0,0)$, then f is continuous at $(0,0)$

(2) If $\frac{\partial f}{\partial x}$ exists at $(0,0)$ then f is continuous separately in both variables x and y at $(0,0)$,

(3) If f is continuous at $(0,0)$, then $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0,0)$

(4) None the above

13. Let $f(x,y) = \frac{(2xy)}{(x^4+y^2)}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$ Then $\lim_{(x,y)\to(0,0)} f(x,y)$

(1) is equal to 0 (2) is equal to 1

(3) does not exist. (4) is equal to ∞

14. Let $f(x,y) = \frac{(xy)}{(x^2+y^2)}$ when $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

Which of the following statements is true?

(1) f is continuous everywhere on the Euclidean plane R^2 apart from (0,0).

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(2) f is continuous everywhere on the Euclidean plan R^2 apart from (0,0) but differentiable nowhere.

(3) f is differentiable everywhere on the Euclidean plane R^2.

(4) f is continuous everywhere on the Euclidean plane R^2 and also differentiable everywhere apart from (0,0).

15. Let $f(x,y) = x\sin(xy) + x^2 + xy + 3$ compute the partial derivatives of f at $(0,1)$.

(1) $\frac{\partial f}{\partial x}|(0,1) = 0$ and $\frac{\partial f}{\partial y}|(0,1) = 0$

(2) $\frac{\partial f}{\partial x}|(0,1) = 1$ and $\frac{\partial f}{\partial y}|(0,1) = 1$

(3) $\frac{\partial f}{\partial x}|(0,1) = 0$ and $\frac{\partial f}{\partial y}|(0,1) = 1$

(4) $\frac{\partial f}{\partial x}|(0,1) = 1$ and $\frac{\partial f}{\partial y}|(0,1) = 0$

16. Examine for maxima and minima:

$f(x,y) = x^2 + 5y^2 - 6x + 10y + 6.$

(1) $F_1(3,1) = -8$ (2) $F_1(3,1) = 8$

(3) $F_1(3,1) = -8$ (4) $F_1(3) = -8$

17. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a mapping such that $f(0,0) = 0$. Determine which of the following is not jointly continuous at $(0,0)$:

(1) $f(x,y) = \frac{x^2y^2}{x^2+y^2}.(x,y) \neq (0,0)$.

(2) $f(x,y) = \frac{xy}{x^2+y^2}, (x,y) \neq (0,0)$.

(3) $f(x,y) = \begin{cases} x\sin\frac{1}{y} + y\sin\frac{1}{x} & if\ xy \neq 0 \\ 0 & otherwise \end{cases}$

(4) None of the above

18. Let $f(x,y) = x^4 - 2x^2y^2 + y^4 + y^4 + x^2 - 6xy + 9y^2$.

Examine whether f admits a local maximum or minimum at (0,0).

(1) minimum (2) maximum

(3) stable (4) none

Answer Key									
1	2	3	4	5	6	7	8	9	10
3	2	4	1	3	4	1	4	1	2
11	12	13	14	15	16	17	18		
4	4	3	1	4	1	4	2		

EXPLANATIONS

[CUET-PG]

1. Explanation:

$u(x,y) = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right) 0 < x < y < 1$

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y)$

$\to \cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{1}{2}\cos u$

$-x\sin u\frac{\partial u}{\partial x} - y\sin u\frac{\partial u}{\partial x} = \frac{1}{2}\cos u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{-1}{x}\frac{\cos x}{\sin u} = \frac{-1}{2}\cot u$

2. Explanation:

1. $F(x,y) = \sqrt{|xy|}$

$F_x = \lim_{n\to 0}\frac{f(nu,0)-f(0,d)}{n}$

$F_x = 0$

$F_y = \lim_{k\to 0}\frac{F(0,ku_1)-f(0,0)}{K}$

$F_y = 0$

F_x & F_y exists at $(0,0)$

2. $\lim_{n\to 0}\frac{f(a+nu_1,b+hu_2)-f(a,b)}{h}$

$\lim_{n\to 0}\frac{f(nu,nu_2)-f(0,0)}{n}$

$\lim_{n\to 0}\frac{\sqrt{n^2u_1u_2}-0}{h} = \lim_{n\to 0}\frac{|n|\sqrt{u_1u_2}}{n}$

$\Rightarrow$ directional derivative does not exists at $(0,0)$ F is not differentiable.

3. Explanation:

$f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & x^2+y^2 \neq 0 \\ 0 & if\ x = y = 0 \end{cases}$

$\lim_{x\to 0}\lim_{y\to 0}F(x,y) = \lim_{x\to 0}\lim_{y\to 0}\frac{x^2-y^2}{x^2+y^2}$

$= \lim_{x\to 0}\frac{x^2}{x^2} = 1$

$\lim_{y\to 0}\lim_{x\to 0}F(x,y) = \lim_{y\to 0}\lim_{x\to 0}\frac{x^2-y^2}{x^2+y^2} = -1$

$\therefore a = 1\ b = -1$

4. Explanation:

BY DIRECT DEFINITION

5. Explanation:

$f(x,y) = \begin{cases} \frac{x}{\sqrt{x^2+y^2}} & x \neq 0, y \neq 0 \\ 2 & x = 0, y = 0 \end{cases}$

$x = r\cos\theta$

$y = r\sin\theta$

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(b) $f(x,y)=\left\{\frac{r\cos\theta}{\sqrt{r^2\cos^2\theta+r^2\sin^2\theta}}\right.$

$\lim_{r\to0} f(x,y)=\lim_{r\to0}\cos\theta=\cos\theta$

Limit depends on θ

$f(x,y)$ is not continuous

$f(x,y)$ not continuous at origin

$f(x,y)$ not differentiable at origin

6. Explanation:

$u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf(x,y)$

$P(u)=\cos u=\frac{x+y}{\sqrt{x}+\sqrt{y}}$

$-x\sin u\frac{\partial u}{\partial x}-y\sin u\frac{\partial u}{\partial x}=\frac{1}{2}\cos u$

$x\sin u\frac{\partial u}{\partial x}+\sin u\frac{\partial u}{\partial x}=-\frac{1}{2}\cot u$

[CU-CET]

1. Explanation:

$f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^2} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$

$\Rightarrow$ choose the $y=r\sin\theta$, $x=r\cos\theta$

$=\lim_{r\to0}\frac{r\cos\theta r^2\sin^2\theta}{r^2\sin^2\theta+r^2\cos^2\theta}$

$=\lim_{r\to0}\frac{r^3\cos\theta\sin^2\theta}{r^2(\sin^2\theta+\cos^2\theta)}$

$=0=f(0)$

$\Rightarrow$ function continuous at $(0,0)$ but not differentiable at (0,0)

$\Rightarrow$ option (1) is correct

2. Explanation:

$F:\mathbb{R}^2\to\mathbb{R}$ given by $F(x,y)=xy$

$f_x=y \qquad f_y=x$

$f_{xx}=0 \qquad f_{yy}=0$

$f_{xy}=1 \qquad f_{yx}=1$

Now,

$(f_{xy})^2-f_{xx}f_{yy}=(1)^2-(0)(0)=1>0$

$\Rightarrow f(0,0)$ is saddle point

$\Rightarrow$ Option c is correct

3. Explanation:

$\alpha=\lim_{(x,y)\to(0,0)}\frac{\sin(x^2+y^2)}{x^2+y^2}$

$\lim_{x\to0}\frac{\sin(x^2)}{x^2}=1$

Ily, $\lim_{y\to0}\frac{\sin(y^2)}{y}=1$

α exist

$\beta=\lim_{(x,y)\to(0,0)}\frac{x^2-y^2}{x^2+y^2}$

$\lim_{x\to0}\frac{x^2}{x^2}=1$

$\lim_{y\to0}\frac{-y^2}{y^2}=-1$

β does not exist

$\Rightarrow$ option (1) is correct

4. Explanation:

$f(x,y)=x^3+y^3-3x-12y+20$

$f_x=3x^2-3$

$f_y=3y^2-12$

$A=f_{xx}=6x$

$B=f_{xy}=0$

$C=f_{yy}=6y$

$D=f_{xx}\cdot f_{yy}\cdot f_{xy}^2$

$6x\cdot6y-0$

$D(-1,2)=6(-1)\times6(2)$

$=-72<0$

Function has neither maxima nor minima if has saddle pt at $(-1,2)$ & $(1,-2)$

$\Rightarrow$ option (4) is correct

5. Explanation:

$f(x,y)=\begin{cases}\frac{xy}{x^2+y^2} & x^2+y^2\neq0\\ 0 & x+y^2=0\end{cases}$

We take path $y=mx$

$\lim_{x\to0}\frac{xmx}{x^2+m^2x^2}=\lim_{x\to0}\frac{mx^2}{x^2+m^2x^2}$

$=\frac{m}{1+m^2}$

Limit depend on m.

Different limit along diff. value of m.

$\Rightarrow$ function is not continuous

$\Rightarrow$ function is not differentiable

$\Rightarrow$ option (3) is correct

6. Explanation:

$f(x)=\begin{cases}x^2+y^2 & x\ \&\ y\ rational\\ 0 & otherwise\end{cases}$

$\lim_{(x,y)\to(0,0)} f(x)=\lim_{(x,y)\to(0,0)} x^2+y^2=0=f(0)$

Function continuous only $(0,0)$ other pt function not diff. & not continuous

$\Rightarrow$ option (2) is correct

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7. **Explanation:**

$f(x,y) = x^y$

$f_x(x,y) = yx^{y-1}$

$f_x(a,0) = 0.a^{0-1} = 0$

$f_y = x^y \log x$

$f_y(e,0) = e^0 \log e = 1$

(3) $f_x = yx^{y-1}$

$f_{xy} = y(y-1)x^{y-2} + x^{y-1}$

$f_{xy}(1,0) = 0 + 1^{-1}$

$= 1$

(4) $f_y = x^y \log x$

$f_{yx} = yx^{y-1} \log x + \frac{x^y}{x}$

$f_{yx}(1,1) = 1.1^0 \log(1) + 1$

$= 1$

$f_{yx}(1,1) = 1$

⇒ option (A) & (D) correct

8. **Explanation:**

$f(x,y)$ is Homogeous function of degree n in $x \subset y$ and has continuous partial derivatives then

$x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y}$ is

$\Rightarrow f(x,y) = x^n + y^n$

$\frac{\partial f}{\partial x} = nx^{n-1}\frac{\partial f}{\partial y} = ny^{n-1}$

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x\,nx^{n-1} + y.ny^{n-1}$

$= nx^{n-1+1} + ny^{n-1+1}$

$= nx^n + ny^n$

$= n(x^n + y^n)$

$= nf(x,y)$

⇒ option (2) is correct

9. **Explanation:**

$\lim_{(x,y)\to(2,1)} (x^2 + 2x - y^2) = 2^2 + 2(2) - 1^2$

$= 4 + 4 - 1$

$= 7$

⇒ Option (3) is correct

10. **Explanation:**

$\lim_{(x-y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$

Let $y = m\sqrt{x}$

$\lim_{(x-y)\to(0,0)} \frac{2xy^2}{x^2+y^4} = \lim_{x\to 0} \frac{2mx^2}{x^2+m^4x^2} = \frac{2m}{1+m^4}$

But limit depends on m

$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4} = DNE$

Option (2) correct.

11. **Explanation:**

$F(x,y) = x^2 + 2xy + y^2$

Here $F(x,y)$ is Homogeneous of degree 2

[PRACTICE QUESTIONS]

1. **Explanation:**

$x = r\sin\theta\cos\phi \;\; y = r\sin\theta\sin\phi \;\; z = r\cos\theta.$

$$\frac{\partial(x,y,z)}{\partial(r\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta} & \frac{\partial x}{\partial\phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta} & \frac{\partial y}{\partial\phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial\theta} & \frac{\partial z}{\partial\phi} \end{vmatrix}$$

$$= \begin{vmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix}$$

$=\backslash sin\ \theta\cos\phi[+r^2\sin^2\theta cos\phi]$

$--r\cos\theta\, cos\phi[-r\sin\theta\cos\theta\cos\phi]$

$-r\sin\theta\sin\phi[-r\sin^2\theta\sin\phi - rcso^2\theta\sin\phi]$

$= r^2\sin^3\theta\cos^2\phi + r^2\sin\theta\cos^2\theta\cos^2\phi$

$+r^2\sin^3\theta\sin^2\phi + r^2\sin\theta\sin^2\phi\cos^2\theta$

$= r^2\sin^3\theta(\sin^2\phi + \cos^2\phi) + r^2\sin\theta\cos^2\theta$

$(\cos^2\phi + \sin^2\phi)$.

$= r^2\sin^3\theta + r^2\sin\theta\cos^2\theta$

$= r^2\sin\theta(\sin^2\theta + \cos^2\theta)$

$= r^2\sin\theta$

⇒ option (3) correct

2. **Explanation:**

$x = r\cos\theta$

$y = r\sin\theta$

$r = \frac{x}{\cos\theta}$

$x^2 + y^2 = r^2$

$\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)}$

$$= \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta} \end{vmatrix} \cdot \begin{vmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial\theta}{\partial x} & \frac{\partial\theta}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} \cdot \begin{vmatrix} \frac{x}{r} & \frac{y}{r} \\ \frac{-y}{r^2} & \frac{x}{r^2} \end{vmatrix}$$

$= r\cos^2\theta + r\sin^2\theta.\frac{x^2}{r^3} + \frac{y^2}{r^3}$

$= r.\frac{x^2+y^2}{r^3}$

$= r.\frac{r^2}{r^3}$

$= 1.$

⇒ option (2) is correct

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3. **Explanation:**

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=3v$$

4. **Explanation:** Contently at $(0,0)$:

Let $x=r\cos\theta, y=r\sin\theta$ then $(x,y)\to(0,0)$ Mean that $r\to 0$

$$\therefore \lim_{(x,y)\to(0,0)} f(x,y)=\lim_{r\to 0}\frac{r^3(\cos^3\theta-\sin^3\theta)}{r^2(\cos^2\theta+\sin^2\theta)}$$

$$=\lim_{r\to 0} r(\cos^3\theta-\sin^3\theta)=0 \text{ as } |\cos^3\theta-\sin^3\theta|\le 2$$

Hence $f(x,y)$ in confinom at (0,0),

Hence option (1) is correct

Also,

$$f_x(0,0)=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{1}{h}\frac{h^3}{h^2}=1$$

And,

$$=\lim_{k\to 0}\frac{-2k(1+2h^2)}{(1+h^2+k^2)^2}=0=\lim_{k\to 0}\frac{1}{k}\frac{-k^3}{k^2}=-1$$

$\therefore f_x(0,0)\neq f_y(0,0)$

Hence option (1) is correct

$$f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^4}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

5. **Explanation:** Continuity at (0,0) :

Let us consider the path $x=my^2$, where we m is arbitray. Then $(x,y)\to(0,0)$ imples $y\to 0$

$$\therefore \lim_{(x,y)\to(0,0)}\frac{xy^2}{x^2+y^4}=\lim_{y\to 0}\frac{my^4}{m^2y^4+y^4}=\frac{m}{1+m^2}$$

Here $\lim_{(x,y)\to(0,0)} f(x,y)$ depends on choice of m hence lint de net exist in general

$\therefore$ f (x, y) is not continuous at (0,0)

Directional derivative:

$$D_u f(0,0)=\lim_{\rho\to 0}\frac{f(0+\rho u_1+\rho u_2)-f(0)}{\rho}$$

$$=\lim_{\rho\to 0}\frac{1(\rho u_1)(\rho u_2)^2}{\rho((\rho u_1)^2+(\rho u_2)^4)}=\lim_{\rho\to 0}\frac{\rho^2 u_1 u_2^2}{\rho^2(u_1^2+\rho^2 u_2^4)}$$

$$=\frac{u_1u_2^2}{u_1^2}$$

$\Rightarrow \frac{u_2^2}{u_1}$ exist for $u_1\neq 0$

For $u_1=0$, it is partial derivative with respect to y

$$\therefore f_y(0,0)=\lim_{k\to 0}\frac{f(0,k)-f(0,0)}{k}=\lim_{k\to 0}\frac{0-0}{k}=0$$

Hence $D_u f(0,0)$ exist for all u

Hence option (3) is cert

6. **Explanation:** $f:\mathbb{R}^2\to\mathbb{R}$ is defined as

$Lf(x,y)=\frac{x^2-y^2}{1+x^2+y^2}$

Note that

$f_{xy}(0,0)=\lim_{k\to 0}\frac{f_y(h,0)-f_y(0,0)}{h}$ and $f_{yx}(0,0)=\lim_{k\to 0}\frac{f_x(0,k)-f_x(0,0)}{k}$

Now

$$f_x(0,0)=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{1}{h}\frac{h^2}{1+h^2}=\lim_{h\to 0}\frac{h}{1+h^2}=0$$

$$f_y(0,0)=\lim_{k\to 0}\frac{f(0,k)-f(0,0)}{k}=\lim_{k\to 0}\frac{1}{k}\frac{-k^2}{1+k^2}=0$$

$$f_x(0,k)=\lim_{h\to 0}\frac{f(h,k)-f(0,k)}{h}=\lim_{h\to 0}\frac{1}{h}\left[\frac{h^2-k^2}{1+h^2+k^2}+\frac{k^2}{1+k^2}\right]$$

Applying L' Hopital rule we have

$$=\lim_{h\to 0}\frac{2h(1+2k^2)}{(1+h^2+k^2)^2}=0$$

$$f_y(h,0)=\lim_{k\to 0}\frac{f(h,k)-f(h,0)}{k}$$

$$=\lim_{k\to 0}\frac{1}{k}\left[\frac{h^2-k^2}{1+h^2+k^2}-\frac{h^2}{1+h^2}\right]\left(\frac{0}{0}\right) \text{ from}$$

Applying L' Hopital rule we have

$$=\lim_{k\to 0}\frac{-2k(1+2h^2)}{(1+h^2+k^2)^2}=0$$

$$\therefore f_{xy}(0,0)=\lim_{h\to 0}\frac{f_y(h,0)-f_y(0,0)}{h}=\lim_{h\to 0}\frac{0-0}{h}=0$$

$$f_{xy}(0,0)=\lim_{k\to 0}\frac{f_x(0,k)-f_x(0,0)}{k}=\lim_{k\to 0}\frac{0-0}{k}=0$$

Hence option (4) is correct

7. **Explanation:** $f(x,y)=\sqrt{|xy|}$

Continuity at (0,0) :

Since,

$$f(0,0=\sqrt{|0|}=0)$$

And

$$\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{(x,y)\to(0,0)}\sqrt{|xy|}=$$

$$\lim_{x\to 0}\sqrt{|x|}.\lim_{y\to 0}\sqrt{|y|}=0$$

$\therefore$ f is continuous at (0,0)

Also

$$\frac{\partial f}{\partial x}(0,0)=\lim_{h\to 0}\frac{f(0+h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{0-0}{h}=0$$

Hence option (1) is comet

8. **Explanation:** $f:\mathbb{R}^2\to\mathbb{R}$ is defined

$$f(x,y)=\begin{cases}\frac{xy}{x^2+y}; & x^2\neq -y\\ 0; & x^2=-y\end{cases}$$

Continuity at (0,0):

Note that the points approaches to the path $x^2=-y$

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The limiting value do not exist, hence $f(x, y)$ is not continuous at (0, 0)

Now

Let $\hat{p} = (P_1, P_2)$ be unit vector, then

$$D_{\hat{p}}f(0,0) = \lim_{\rho\to 0}\frac{f(0+\rho p_1, 0+\rho p_2) - f(0,0)}{\rho}$$

$$= \lim_{\rho\to 0}\frac{1}{\rho}\left(\frac{\rho p_1 \cdot \rho p_2}{(\rho p_1)^2+\rho p_2} - 0\right) = \lim_{\rho\to 0}\frac{p_1 p_2}{\rho p_1+\rho p_2} = \frac{p_1 p_2}{p_2} = p_1$$

Hence exist for all vector $\hat{p} = (P_1, P_2)$

Hence option (4) is correct

9. Explanation: Given that,

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}\ ; & (x,y) \neq (0,0) \\ 0\ ; & (x,y) = (0,0) \end{cases}$$

Continuity at (0, 0) :

Let us consider the path $y = mx^2$, then, $(x, y) \to (0, 0)$ implies that $x \to 0$

$$\therefore \lim_{(x,y)\to(0,0)}\frac{x^2y}{x^4+y^2} = \lim_{x\to 0}\frac{x^2.mx^2}{x^4+(mx^2)^2} = \lim_{x\to 0}\frac{mx^4}{(1+m^2)x^4} = \frac{m}{1+m^2}$$

Here $\lim_{(x,y)\to(0,0)} f(x, y)$ depends on the choice of a hence it does not exist at (0,0)

∴ f is not continuous at (0,0)

Directional derivative at (0, 0) :

Let $\hat{p} = p_1\hat{i} + p_2\hat{j}$ be arbitrary unit vector

Then

$$D_{\hat{p}}f(0,0) = \lim_{\rho\to 0}\frac{f(0+\rho p_1.0+\rho p_2)-f(0,0)}{\rho} = \lim_{\rho\to}\frac{1}{\rho}\frac{(\rho p_1)^2(\rho p_2)}{(\rho p_1)^4+(\rho p_2)^2}$$

$$\Rightarrow \lim_{\rho\to 0}\frac{\rho^2(p_1^2p_2)}{\rho^2(\rho^2p_1^4+p_2^2)} = \frac{p_1^2p_2}{p_2^2}$$

$\Rightarrow \frac{p_1^2}{p_2}$ exists if $p_2 \neq 0$

If $p_2 = 0$, then directional derivative is same as partial derivative with respect to x at (0, 0). And

$$f_x(0,0) = \lim_{h\to 0}\frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0}\frac{0-0}{h} = 0 \text{ exits}$$

∴ directional derivative exist in all the direction at (0, 0)

Hence option (1) is correct

10. Explanation: Consider the action

$f(x, y) = 4y - x^2$

Then grad $f = f_x\hat{i} + f_y\hat{j} = (-2x)\hat{i} + 4\hat{j}$

∴ grad $f|_{(1,5)} = -2\hat{i} + 4\hat{j}$

direction is given as $\bar{v} = (4-1)\hat{i} + (3-5)\hat{j} = 3\hat{i} - 2\hat{j}$

$$\Rightarrow \hat{v} = \frac{\bar{v}}{|\bar{v}|} = \frac{3\hat{i}-2\hat{j}}{\sqrt{9+4}} = \frac{3}{\sqrt{13}}\hat{i} - \frac{2}{\sqrt{13}}\hat{j}$$

$$\therefore D_{\hat{v}}f|_{(1,5)} = (\text{gradf})\left(\frac{3}{\sqrt{13}}\hat{i} - \frac{2}{\sqrt{13}}\hat{j}\right) = (-2)\left(\frac{3}{\sqrt{13}}\right) + 4\left(\frac{-2}{\sqrt{13}}\right) = \frac{-14}{\sqrt{13}}$$

Hence correct options (2)

11. Explanation: Given that

$f : \mathbb{R}^2 \to \mathbb{R}$ as $f(x, y) = |x| + |y|$

$\therefore f(0,0) = |0| + |0| = 0 + 0 = 0$

And

$$\lim_{(x,y)\to(0,0)}(|x| + |y|) = \lim_{x\to 0}|x| + \lim_{y\to 0}|y| = 0 + 0 = 0$$

∴ $f(x, y)$ is continuous at (0, 0)

Also

$f_x(0,0) = \lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0}\frac{|h|-0}{h}$ do not exists

Similarly

$f_y(0,0) = \lim_{k\to 0}\frac{|k|}{k}$ do not exist

Hence option (4) is correct

12. Explanation:

a) Let $F(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

$$F_x(0,0) = \lim_{n\to 0}\frac{F(h,0) - F(0,0)}{n}$$

$$= \lim_{n\to 0}\frac{\frac{n.0}{n^2+0^2} - 0}{n} = 0$$

$\Rightarrow F_x$ exist

$$F_y(0,0) = \lim_{k\to 0}\frac{F(0,k) - F(0,0)}{k}$$

$$= \lim_{k\to 0}\frac{\frac{0.k}{0^2+k^2}}{k} = 0$$

$\Rightarrow F_y$ exists

If $y = mx$ then,

$$\lim_{(x,y)\to(0,0)} F(x, y) = \lim_{x\to 0} F(x, mx)$$

$$= \lim_{x\to 0}\frac{x^2m}{x^2+x^2m^2} = \frac{m}{1+m^2}$$

Limit depend on path

$\Rightarrow F(x, y)$ not continuous

$\Rightarrow$ option (1) is wrong

b) Now, $= F(x, y) = \frac{(x+1)y}{x^2+y^2}$

$$F_x = \lim_{n\to 0}\frac{F(n,0) - F(0,0)}{n}$$

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$$= \lim_{n\to 0} \frac{\frac{(n+1)0}{n^2+0^2}}{n} = 0$$

$\Rightarrow F_x$ exist

Now, $\lim_{x\to 0} \frac{F(x,y)}{x^2+y^2} = \frac{(0+1)y}{0+y^2}$

$= \frac{y}{y^2} = \frac{1}{y}$

$F_y = \frac{1}{y}$

$\Rightarrow$ F is not continuous separately at (x, y) = 0

$\Rightarrow$ option (2) is correct

13. Explanation:

$$f(x,y) = \begin{cases} \frac{2xy}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{2xy}{x^4+y^2}$

$x = r\cos\theta, y = r\sin\theta$

$\lim_{r\to 0} f(x,y) = \lim_{r\to 0} \frac{2r^2\cos\theta+\sin\theta}{x^4\cos^4\theta+r^2\sin^2\theta}$

$= \lim_{r\to 0} \frac{\sin 2\theta}{r^2\cos^4\theta+\sin^2\theta}$

$= \lim_{r\to 0} \frac{2\sin\theta\cos\theta}{r^2\cos^4\theta+\sin^2\theta}$

$= \frac{2\cos\theta}{\sin\theta}$

$= 2\cot\theta$

Limit depends on θ

$\Rightarrow \lim_{(x,y)\to(0,0)} f(x,y)$ DNE

14. Explanation:

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2y^2}$

Let $x = r\cos\theta\ y = r\sin\theta$

$\lim_{r\to 0} f(x,\theta) = \lim_{r\to 0} \frac{r^2\sin\theta\cos\theta}{r^2} = \lim_{r\to 0} \sin\theta - \cos\theta =$

$\sin\theta\cos\theta$

Limit depends on θ

$\Rightarrow f$ is not continuous

15. Explanation:

Let $f(x,y) = x\sin(x,y) + x^2 + xy + 3$

$\frac{\partial f}{\partial x} = \sin(x,y) + yx\cos(x,y) + 2x + y$

$\left.\frac{\partial f}{\partial x}\right|_{(0,1)} = 1$

$\frac{\partial f}{\partial y} = x^2\cos(xy) + x$

$\left.\frac{\partial f}{\partial x}\right|_{(0,1)} = 0$

16. Explanation:

$f(x,y) = x^2 + 5y^2 - 6x + 10y + 6$

$H = \begin{bmatrix} f_{xx} & fxy \\ fxy & fyy \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix}$

$|H| = 20 > 0$

& $F_{xn} > 0$

$\Rightarrow$ local minima.

$F_2 = 2x - 6 = 0 \qquad F_y = 10y + 10 = 0$

$\Rightarrow x = 3 \qquad y = -1$

$F_1(3,1) = 9 + 5 - 18 + 10 - 5$

$F_1(3,1) = -8$

17. Explanation:

1. $F(x,4) = \frac{x^2y^2}{x^2-y^2}$

Consider, $y = mx$

$\lim_{x\to\infty} F(x,mx) = \lim_{x\to\infty} \frac{x^2(m^2x^2)}{x^2+(mx)^2}$

$= \lim_{x\to\infty} \frac{x^4m^2}{x^2+(1+m^2)}$

$\lim_{x\to\infty} F(x,y) = 0 = F(0,0)$

$\Rightarrow F(x,y)$ is continuous

2. $F(x,y) = \frac{xy}{x^2+y^2}$

Consider Path

$y = mx$

$\lim_{x\to\infty} F(x,mx) = \lim_{x\to\infty} \frac{m\,x^2}{x^2+m^2\,x^2} = \frac{m}{1+m^2}$

$= \lim_{(x,y)\to(0,0)} F(x,y)$ depends on m

$\Rightarrow F(x,y)$ is continuous

3. $F(x,y) = x\sin\frac{1}{y} + y\sin\ \frac{1}{x}$

$\lim_{(x,y)\to(0,0)} F(x,y) = \lim_{(x,y)\to(0,0)} = \left(x\sin\frac{1}{y} + y\sin\frac{1}{x}\right)$

Consider Path

$y = mx$

$\lim_{x\to\infty} x\sin\frac{1}{mx} + mx\sin\left(\frac{1}{x}\right) = 0 = F(0,0)$

$\Rightarrow F(x,y)$ is continuous

18. Explanation:

$F(x_1y) = x^4 - 2x^2y^2 + y^4 + x^2 - 6xy + 9y^2$

$H = \begin{bmatrix} 12x^2 - 8y^2 + 2 & -8xy - 6 \\ -8xy - 6 & -4x^2 + 12y^2 + 18 \end{bmatrix}$

$x = 0 \qquad y = 0$

$H = \begin{bmatrix} 2 & -6 \\ -6 & +8 \end{bmatrix}$

$|H| = 16 - 36 = -20 < 0$

(0,0) is maximum

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